average density is approaching 225 cars per mile, an amount much less than that associated with the local traffic conditions near the 50 mark.

For a reasonable *local* value of density to exist we must assume that the distance between cars stays approximately the same over a distance that includes many cars. From this example, we see this is not always entirely the case (in particular, this situation gets worse for less dense traffic conditions). However, in this text we will assume that a local traffic density is a reasonable variable.

Alternatively, especially when an appropriate measuring interval does not exist, a statistical theory may be formulated. We will limit our investigation to deterministic models.

In this text we will primarily be concerned with traffic situations involving a large number of vehicles. We will mostly be interested in the collective behavior of many vehicles (a macroscopic theory). For this reason we will be satisfied with investigating the behavior of average traffic variables (i.e., density and flow). We will assume measuring intervals exist (in space and time) such that the traffic density and traffic flow are smooth functions of position and time without the local variations being lost as a result of an extremely long measuring interval. Neglecting the possibly rapid variation from car to car of car-following distance and thus utilizing the concept of a smooth traffic density will yield a description of traffic flow problems which hopefully will be a good model. This is called the **continuum hypothesis.** (It is also used in fluid dynamics. Can you guess where the continuum hypothesis arises in investigating motions of liquids and gases?)

EXERCISES

58.1. If at 10 m.p.h. (16 k.p.h.) cars are one car-length behind each other, then what is the density of traffic? [You may assume that the average length of a car is approximately 16 feet (5 meters)].

58.2. Assume that the probability P of exactly one car being located in any fixed short segment of highway of length Δx is approximately proportional to the length, $P = \lambda \Delta x$. Also assume that the probability of two or more cars in that segment of highway is negligible.

(a) Show that the probability of there being exactly n vehicles along a highway of length x, $P_n(x)$, satisfies the **Poisson distribution,**

$$P_n(x) = \frac{(\lambda x)^n e^{-\lambda x}}{n!}.$$

[Hint: Consider $P_n(x + \Delta x)$ and form a differential equation for $P_n(x)$, (see Sec. 36).]

(b) Evaluate and interpret the following quantities:

(1) $P_n(0)$, $n \neq 0$.

(2) $P_0(x)$, $x \neq 0$.

(3) $P_1(x)$, $x \neq 0$.

(c) Calculate the expected number of cars on a highway of length x. Interpret your answer.

58.3. Consider the example discussed in this section in which the density is calculated as a function of the measuring interval. The density has a discontinuity at distances that first include an additional car.

(a) What is the jump in density (i.e., the magnitude of the discontinuity)?

(b) If cars are equally spaced (100 per mile), how large must the measuring interval be such that discontinuities in density are less than 5 percent of the density? How many cars are contained in that measuring distance?

(c) Generalize your result to a roadway with a constant density of ρ_0 per mile.

59. Flow Equals Density Times Velocity

In the past sections we have briefly discussed the three fundamental traffic variables: velocity, density, and flow. We will show that there is a close relationship among these three. We first consider one of the simplest possible traffic situations. Suppose that on some road, traffic is moving at a constant velocity u_0 with a constant density ρ_0, as shown in Fig. 59-1. Since each car

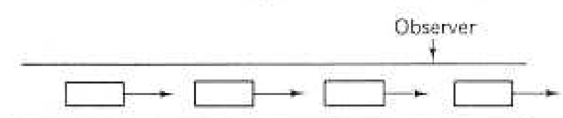

Figure 59-1 Constant flow of cars.

moves at the same speed, the distance between cars remains constant. Hence the traffic density does not change. What is the flow of cars? To answer that, consider an observer measuring the traffic flow (the number of cars per hour that pass the observer). In τ hours each car has moved $u_0\tau$ distance (moving at a constant velocity, the distance travelled equals the velocity multiplied by the time), and thus the number of cars that pass the observer in τ hours is the number of cars in $u_0\tau$ distance, see Fig. 59-2. Since ρ_0 is the number of cars

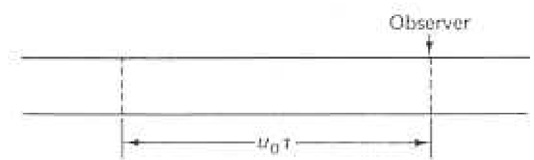

Figure 59-2 Distance a car, moving at constant velocity u_0, travels in τ hours.