E. Kreyszig, Advanced Engineering Mathematics, 9^{th} ed.	Section 7.3, pgs. 287-295
Lecture: The Row Reduction Algorithm	Module: 03
Suggested Problem Set: Suggested Problems : {1, 4, 8, 16 }	January 22, 2010

E. Kreyszig, Advanced Engineering Mathematics, 9^{th} ed.	Section 7.5, pgs. 302-305
Lecture: Solutions of Linear Systems	Module: 03
Suggested Problem Set: Suggested Problems : {Null }	January 22, 2010

Quote of Lecture 3	
Cole and Macey lost their eyes to the finer points. Roll them up in coffee cake and dine.	
	The Shins - Your Algebra (2001)

Last time we studied the algebra of matrices. Specifically, we showed that matrix multiplication is defined in such a way that permits us to write down, in a compact way, a general linear system of equations. We would now like to study how to solve such systems through operations applied to this compact notation. These operations are often called the rules of row reduction and are transcribed directly from algebraic techniques you have been exposed to before. This gives us three rules, which we can apply to linear systems in augmented matrix form. Thought, the application of these three rules does change the augmented matrix it **does not** change the solution to the linear system. The rules of row reduction are:

1. Row Interchanged - Any two rows of an augmented matrix can have their positions interchanged.
2. Row Scaling - Any row of an augmented matrix can be multiplied by a non-zero scalar.
3. Row Replacement - Any row of an augmented matrix can be replaced by itself summed with a scalar multiple of another row.

Applying these three rules we can see by extending concepts from $\mathbb{R}$ and $\mathbb{R}^2$ that there are three fundamental types of solution sets for a given linear system $\mathbf{Ax} = \mathbf{b}$. They are:

1. Unique Solutions - A solution set with exactly one element. See Exercise 1
2. Nonunique Solutions - A solution set with an infinite number of elements. See Example 1 and 2
3. No Solutions - An empty solution set. See Example 3

See the handout entitled 'Three Planes in Space' for visualizations of examples from class that illustrate the previous cases.

Goals

- Develop a correspondence between systems of linear equations and the matrix equation $\mathbf{Ax} = \mathbf{b}$.
- Understand how the the existence and uniqueness of solutions to $\mathbf{Ax} = \mathbf{b}$ can be found by application of the row-reduction algorithm.

Objectives

- Explicitly show that matrix multiplication is defined so that a system of linear equations is equivalent to $\mathbf{Ax} = \mathbf{b}$.
- Motivate row-reduction through previous substitution methods and formalize a procedure of solving $\mathbf{Ax} = \mathbf{b}$ with it.
- Using row-reduction algorithm analyze three prototypical examples concerning existence and uniqueness of solutions to $\mathbf{Ax} = \mathbf{b}$ and when applicable understand the geometry of the solution(s).