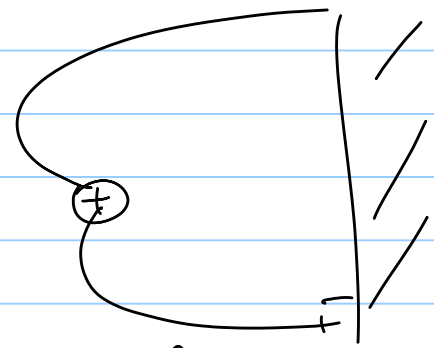
Note Title 2/2/2009

Solve $\nabla^2 V = -\rho/\epsilon_0$

- method images



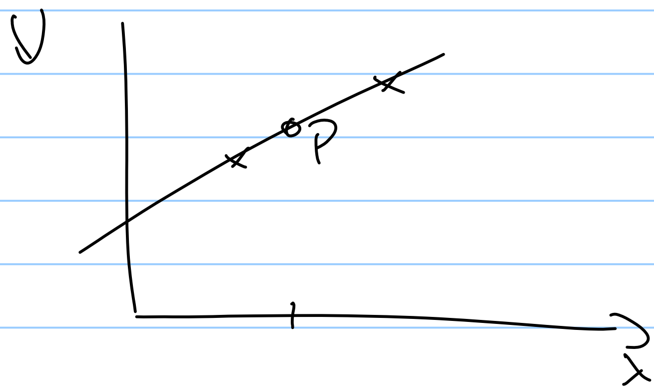

most of space $\rho = 0$

Laplace's eqn

$$\nabla^2 V = 0$$

1-D $\qquad \dfrac{d^2V}{dx^2} = 0 \qquad V(x) = mx + b$

const




(1) $V(P)$ = average value of $V$ in the neighborhood of $P$

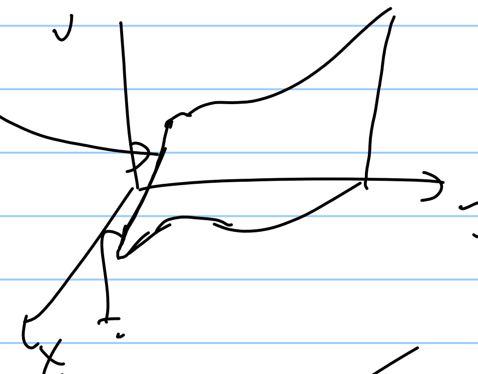
(2) $V$ can have no local maxima or minima

Extreme value of $V$ occur at boundaries

2-D

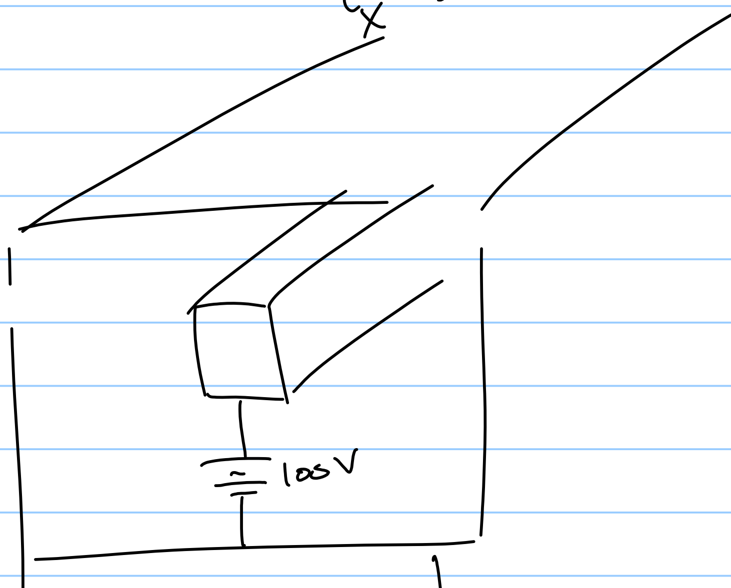
$$\frac{\partial^2}{\partial x^2}V(x,y) + \frac{\partial^2 V(x,y)}{\partial y^2} = 0$$

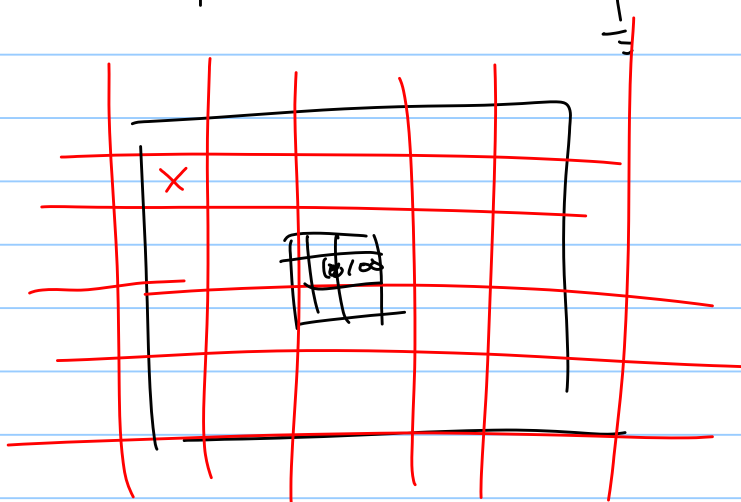
bndry is line which has $\infty$ # pts




Hmwk

Solve $\nabla^2 V = 0$ using relaxation method



Separation of Variables.

$$\nabla^2 V = 0 = \frac{\partial^2 V}{\partial x^2} + \frac{\partial V}{\partial y^2} + -$$

$$V = X(x)\, Y(y)\, Z(z)$$

$$\frac{\frac{\cancel{d}^2 X(x)}{\cancel{d}x^2}\cancel{Y}\cancel{Z}}{X\cancel{Y}\cancel{Z}} + \frac{\frac{\cancel{d}^2 Y}{\cancel{d}y^2}\cancel{X}\cancel{Z}}{\cancel{X}Y\cancel{Z}} + \cdots = 0$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = 0$$

$$\begin{array}{ccccccc} \| & & \| & & \| & & \\ C_1 & + & C_2 & + & C_3 & = & 0 \end{array}$$

$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2 \qquad \frac{1}{Z}\frac{d^2Z}{dz^2} = C_3$$

3 ODE's

Soln depend $C < 0$ $\quad \frac{d^2X}{dx^2} = -|C_1| X$

$$X = A\sin kx + B\cos kx$$

$C = 0$ $\quad X = mx + b$

$C > 0$ $\quad X = A' e^{\sqrt{c}\,x} + B' e^{-\sqrt{c}\,x}$