Linear antennas

small dipole sources don't radiate effectively.
must have $d \sim \lambda$ for both transmission + reception.
e.g. @ 1.9 GHz (cell phones) $\lambda \sim 15$ cm.

types: center driven $\rightarrow$ end driven

center-driven current

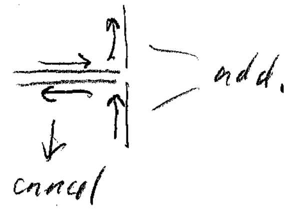
$$\vec{J}(\vec{r}';t)\,d^3r' \longrightarrow I(z',t)\,dz' = \hat{z}\,I_0\,e^{-i\omega t_r} f(z)\,dz$$

thin wire

current has to be zero at ends
$\rightarrow$ standing waves

$$f(z') = \sin k\left(\frac{d}{2} - |z'|\right)$$

note $k = 2\pi/\lambda$ λ is not fixed.
$I(\pm d/2) = 0$
$I(0) = \sin(kd/2)\,e^{-i\omega t_r}$



calc radiation fields using B-field.
depends only on $\partial_t \vec{J}$

$$\vec{B}_{rad} = \int_V \frac{[\partial_t \vec{J}] \times \hat{R}}{c^2 R}\,d^3r' \rightarrow \frac{(\hat{z} \times \hat{r})(-i\omega)}{c^2 r}\int I(z',t_r)\,dz'$$

$R \rightarrow r$ $\hat{R} \rightarrow \hat{r}$

assumptions $r \gg \lambda$ (radiation) $r \gg d$ ("paraxial") we can calc $\vec{E}_{rad} = -\hat{r} \times \vec{B}_{rad}$, but really want radiated power:

$$\vec{S}_{rad} = \frac{c}{4\pi} \vec{E}_{rad} \times \vec{H}_{rad} \rightarrow \frac{c}{4\pi} B_{rad}^2 \hat{r}$$

Key part of integral is $e^{-i\omega t_r}$ term.

- each current element contributes w/ a phase that depends on $t_r = t - |\vec{r} - \vec{r}'|/c$

$$\int I(z') e^{-i\omega t_r} dz' \rightarrow e^{-i\omega t} \int I(z') e^{i\omega|\vec{r}-\vec{r}'|/c} dz'$$

Now integral is over space. Can't set $|\vec{r} - \vec{r}'| \sim r$ b/c phase is rapidly varying.

$\therefore$ expand w/ r'/r small

$$|\vec{r} - \vec{r}'| = \left(r^2 - 2\vec{r}\cdot\vec{r}' + r'^2 \right)^{1/2}$$
$$= r \left(1 - \frac{2\hat{r}\cdot\vec{r}'}{r} + (r'/r)^2 \right)^{1/2}$$

$$\simeq r \left[1 - \frac{\hat{r}\cdot\vec{r}'}{r} + \frac{1}{2}\left(\frac{r'}{r}\right)^2 - \frac{1}{8}\left(\frac{2\hat{r}\cdot\vec{r}'}{r}\right)^2 + O\left(r'/r\right)^3 \right]$$

$$\doteq r \left[1 - \frac{r'}{r}\cos\theta + \frac{1}{2}\left(\frac{r'}{r}\right)^2 - \frac{1}{2}\left(\frac{r'}{r}\right)^2 \cos^2\theta \right)$$

$$= r \left(1 - \frac{r'}{r}\cos\theta + \frac{1}{2}\left(\frac{r'}{r}\right)^2 \sin^2\theta \right)$$

check scale $\quad r' \sim d/2$

$k r' \sim 2\pi \frac{d}{2\lambda}$

$\frac{k}{2}\frac{(r')^2}{r} \sim \frac{2\pi}{2} \frac{d^2}{4r\lambda} \rightarrow$ if $\frac{d^2}{8r\lambda} \ll 1$

or $r \gg d^2/8\lambda$

can drop quadratic term.

this is Fraunhofer limit (shows up again in diffraction)
defines "far-field"

$$B_{rad} \doteq -\frac{i\omega}{c^2 r} \underbrace{\hat{z}\times\hat{r}}_{\sin\theta} e^{i(kr-\omega t)} I_0 \int_{-d/2}^{d/2} \sin k\left(\frac{d}{2}-|z'|\right) e^{ikz'\cos\theta} dz'$$

do integral on Mathematica

$$B_{rad} = -\frac{2I_0}{c} i \frac{e^{i(kr-\omega t)}}{r} \left(\frac{\cos\left[\frac{kd}{2}\cos\theta\right] - \cos kd/2}{\sin\theta} \right)$$

$$\text{calc } \left\langle \frac{dP}{d\Omega} \right\rangle = r^2 \langle \vec{S}_{rad} \rangle \cdot \hat{n} = \frac{cr^2}{4\pi} \langle B_{rad}^2 \rangle$$

Notice that the far field is actually a Fourier transform:

$$G(\beta) = \int_{-\infty}^{\infty} \underbrace{\text{rect}\left(z'/(d/2)\right) \sin k\left(\frac{d}{2}-|z'|\right)}_{g(z)} e^{-i\beta z'} dz'$$

$\beta = k\cos\theta$ is a type of spatial frequency.
units $1/\text{length}$.

Also note wave is a modulated spherical wave: $\sin\theta \frac{e^{i(kr-\omega t)}}{r}$
this is like a dipole radiator -
similar to diffraction.