Physical and Fourier Optics (PHGN570) posted 10 January 2014
Homework 1 due in class 21 January 2014

1. Use Mathematica functions to perform a set of Fourier transform pairs. For the first three, chose t_0=1 and plot the functions in both t and ω domains.
   a. $\Im\left\{\text{rect}\left[t/t_0\right]\right\}$ UnitBox[]
   b. $\Im\left\{\exp\left[-t^2/t_0^{\,2}\right]\right\}$
   c. $\Im\left\{\Lambda\left[t/t_0\right]\right\}$ UnitTriangle[] For this triangle function, also calculate the result by doing the integral directly manually.
   d. $F[\omega]=\Im\left\{\cos\left[\omega_0 t\right]^2\right\}$ The DiracDelta[ ] functions will not plot. Instead, use the function Convolve[ ] to convolve the result F[ω] with a narrow Gaussian pulse: $\exp\left[-\omega^2/d\omega^2\right]\otimes F\left[\omega\right]$ In Mathematica, this is $\rightarrow G\left[\omega\_,d\omega\_\right]=\text{Convolve}\left[\exp\left[-\omega p^2/d\omega^2\right],F\left[\omega p\right],\omega p,\omega\right]$, where ωp is a dummy variable. Make a plot of the t and ω space results this way, choosing the Gaussian width dω for narrow spikes.

2. Symmetry properties of Fourier transforms
   a. Show that if *f(t)* is odd and real, *F(ω)* is imaginary and odd.
   b. Show that if *f(t)* is real, *F(ω)* is in general complex, but is constrained by the symmetry property $F(-\omega)=F^*(\omega)$
   c. Show that when is *f(t)* is real, $\left|F(\omega)\right|^2$ is an even function of ω.
3. Consider a pulse of the form
   $f(t)=Ae^{-|t|/b}e^{-i\omega_0 t}$
   where b is a real, positive constant.
   a. Calculate the Fourier transform $F(\omega)=\Im\{f(t)\}$ by direct integration, manually.
   b. Do this transform using the FourierTransform[ ] function in Mathematica. Our convention for the transforms requires you use the option $\text{FourierParameters}\rightarrow\{1,1\}$.
   c. Now consider the one-sided exponential decay function, letting $l(t)=e^{-t}$ for $t>0$, and $=0$ for $t<0$. Calculate the Fourier transform L(ω) of the decaying exponential function, l. Express $f(t)$ in terms of $l(t)$, and use Fourier identities to calculate F(ω) (no additional integration needed).
   d. Show that for this pulse $\int_{-\infty}^{\infty}\left|f(t)\right|^2 dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|F(\omega)\right|^2 d\omega$ by calculating both sides of the equation, confirming Parseval's theorem. You may use Mathematica for this.

4. Given that the Fourier transform of *f(t)* is *F(ω)*, find general expressions for the Fourier transforms of $g(t) = \int_0^t f(t')dt'$ and $h(t) = df/dt$ in terms of *F(ω)*. Hint: replace *f(t)* with its transform inside the expressions above.