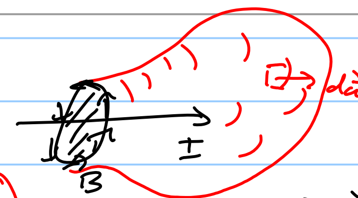
Maxwells eqns:

Note Title 6/19/2006

① $\vec\nabla\cdot\vec E = \frac{\rho}{\epsilon_0}$ ② $\vec\nabla\cdot\vec B = 0$



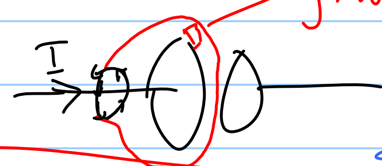
③ $\boxed{\vec\nabla\times\vec E = -\frac{\partial \vec B}{\partial t}}$ ④ $\vec\nabla\times\vec B = +\mu_0\vec J + $

$$\int \vec\nabla\times\vec E\cdot d\vec a = \underbrace{\oint \vec E\cdot d\vec l}_{\mathcal{E}mf} = -\frac{\partial}{\partial t}\underbrace{\int B\cdot da}_{\Phi_m}$$

$$\int \vec\nabla\times\vec B\cdot d\vec a = \oint \vec B\cdot d\vec l$$

$$\int +\mu_0\vec J\cdot d\vec a = +\mu_0 I$$

$\int \mu_0\vec J\cdot d\vec a$



stokes thm

not s.s.

$$\int \vec B\cdot d\vec l = \mu_0 I + \int \mu_0\epsilon_0\frac{\partial \vec E}{\partial t}\cdot da$$

steady state

$$\vec\nabla\times\vec B = \mu_0\vec J + \mu_0\epsilon_0\frac{\partial\vec E}{\partial t}$$

for all $\vec B$

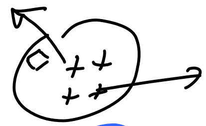
$$\vec\nabla\cdot\underbrace{(\vec\nabla\times\vec B)} = 0 = \mu_0\overset{-\frac{\partial\rho}{\partial t}}{\vec\nabla\cdot\vec J} + \mu_0\epsilon_0\frac{\partial}{\partial t}\overset{\rho/\epsilon_0}{\vec\nabla\cdot\vec E} \equiv 0$$

Cartesian $\vec\nabla\cdot\vec C = \frac{\partial}{\partial x}C_x + \frac{\partial}{\partial y}C_y + \frac{\partial}{\partial z}C_z = 0$

$$\boxed{\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}}$$

conservation of charge

$\frac{C}{s}$ flowing out of volume



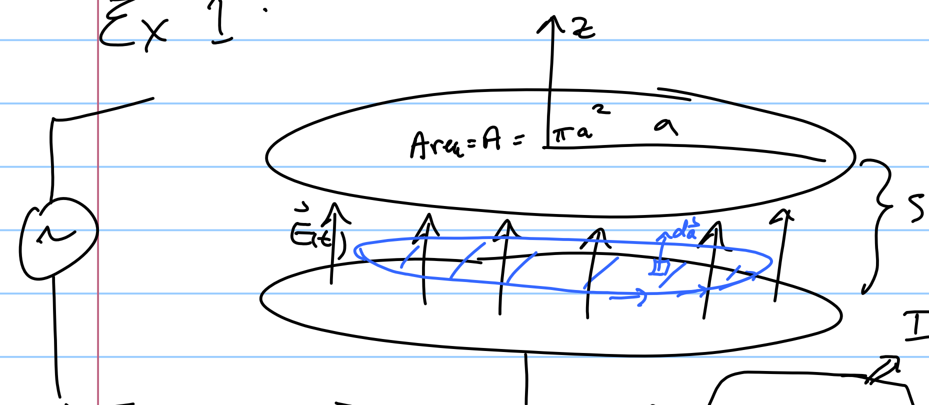
$$\int \vec{\nabla}\cdot\vec{J}\,d\tau = \oint \vec{J}\cdot d\vec{a} = -\frac{dQ_{enc}}{dt} = -\frac{d}{dt}\int \rho\, d\tau = \int -\frac{\partial \rho}{\partial t}\, d\tau$$

$\vec{J} = \rho v$ $\frac{C}{s\, m^2}$

$\frac{C}{m^3}$ $\frac{m}{s}$

- Maxwell's eqns which give $\vec{E}$ & $\vec{B}$
- conservation of charge
- $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B} = m\vec{a}$

Ex 1.



$$I = \epsilon_0 A \frac{\partial E}{\partial t}$$

$\vec{J}_d = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$; $E = \frac{\sigma}{\epsilon_0} = \frac{1}{\epsilon_0}\frac{Q}{A}$; $\boxed{\frac{\partial E}{\partial t} = \frac{1}{\epsilon_0}\frac{I}{A}}$; $\vec{J}_D = \frac{I}{A}\hat{z}$

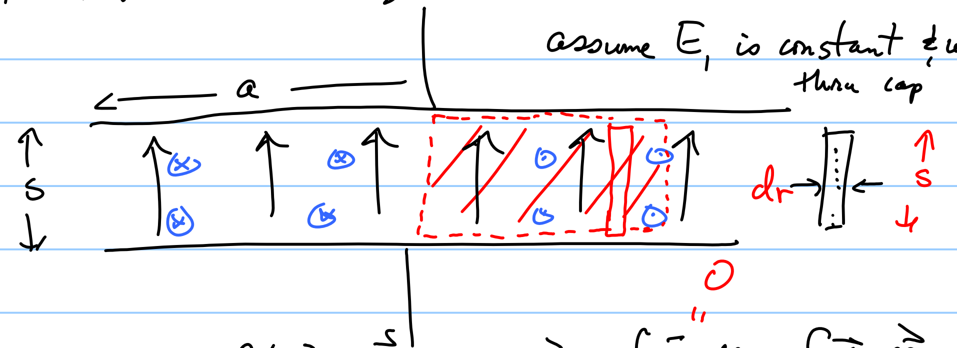
$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \int \vec{J}_d \cdot d\vec{a} = \mu_0 \frac{I}{\pi a^2} \pi r^2 \qquad \boxed{\vec{B} = \frac{\mu_0 I}{2\pi} \frac{r}{a^2} \hat{\phi}}$$

Changing $\vec{E}$ creates $\vec{B}$ (like Faraday's law)

We have $I(t)$ so $B(t)$ & changing B must generate an E.

Approach is iterative soln.

assume E_1 is constant & uniform thru cap



$$\oint \vec{E}\cdot d\vec{\ell} = \oint (\vec{E}_1 + \vec{E}_2 + \cdots)\cdot d\vec{\ell} = \oint \vec{E}_1 \cdot d\vec{\ell} + \oint \vec{E}_2 \cdot d\vec{\ell}$$

0

define $\boxed{E(r=0) = E_0 e^{i\omega t}} = E_1 + E_2$

$E_2(r=0) = 0$

$$\oint \vec{E}\cdot d\vec{\ell} = \oint \vec{E}_2 \cdot d\vec{\ell} = -E_2 s = -\frac{\partial}{\partial t}\Phi_m = -\frac{\partial}{\partial t}\int B_1(r)\, da$$

$$= -\frac{\partial}{\partial t}\int B_1(r)\, s\,dr = -\frac{\partial}{\partial t}\int_0^a \frac{\mu_0 I}{2\pi}\frac{r}{a^2}\, s\,dr$$

$$I = \epsilon_0 A \frac{\partial E}{\partial t} = \epsilon_0 A i\omega E_0 e^{i\omega t}$$

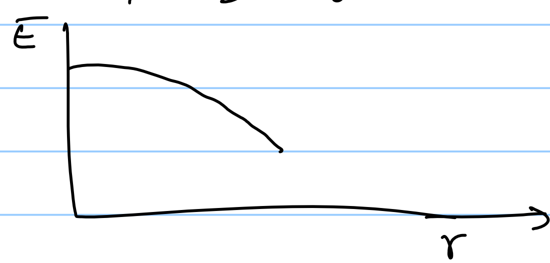
B has a phase shift

$$B_1 = \frac{\mu_0}{2\pi}\frac{r}{a^2}\,\underbrace{\epsilon_0 \underset{\pi a^2}{A}\, i\omega E_0 e^{i\omega t}}_{I} = \frac{i\omega}{2}\mu_0\epsilon_0 r E_0 e^{i\omega t}$$

$$-E_2 \cancel{s} = -\frac{\partial}{\partial t}\int \frac{i\omega}{2}\mu_0\epsilon_0 r E_0 e^{i\omega t}\,\cancel{s}\,dr$$

$$E_2 = -\frac{\omega^2 r^2}{4}\mu_0\epsilon_0 E_0 e^{+i\omega t} \qquad c^2 = \frac{1}{\mu_0\epsilon_0}$$

$$E = E_1 + E_2 = E_0 e^{i\omega t}\left(1 - \frac{1}{4}\frac{\omega^2 r^2}{c^2}\right)$$



Now the mag. field we cal. isn't right because the changing E used to get B was wrong. $B = B_1 + B_2$

where $B_1 = \frac{i\omega}{2}\frac{r}{c^2}E_0 e^{i\omega t}$ due to variations in E_1

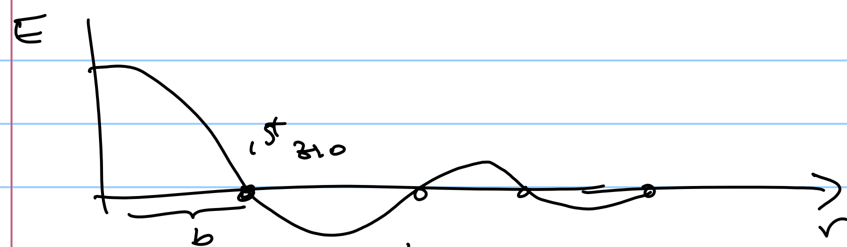
;

B_2 is due to variations in E_2. Go back to 1^{st} figure and redo calculation with just E_2 to get B_2

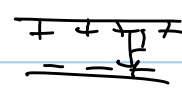
$$E = E_0 e^{i\omega t}\left[1 - \frac{1}{2^2}\left(\frac{\omega r}{c}\right)^2 + \frac{1}{2^2 4^2}\left(\frac{\omega r}{c}\right)^4 + \cdots\right]$$

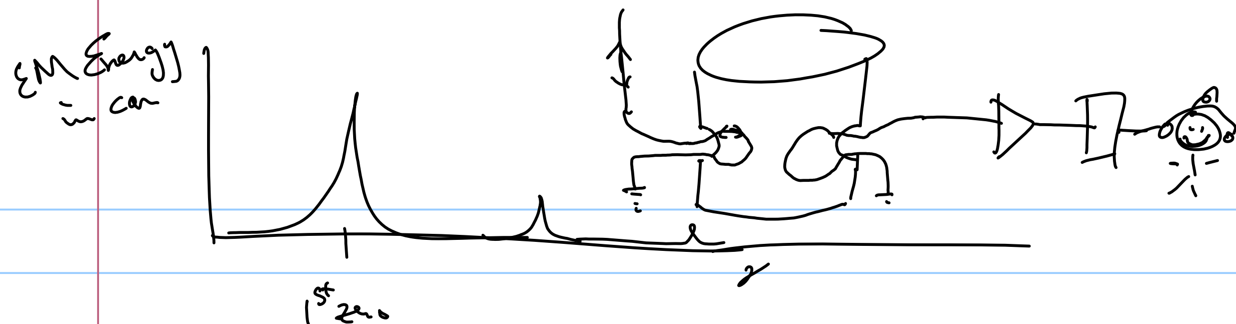
dimesionless

$$J_0(x) = 1 - \frac{1}{(1!)^2}\left(\frac{x}{2}\right)^2 + \frac{1}{(2!)^2}\left(\frac{x}{2}\right)^4 - \frac{1}{(3!)^2}\left(\frac{x}{2}\right)^6 + \cdots$$

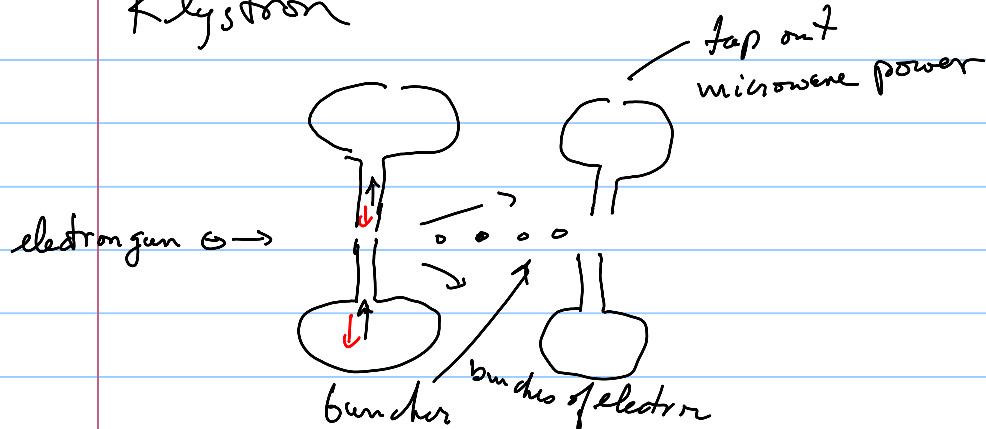


beer can




EM Energy
in cav
1st zero

Klystron


tap out
microwave power
electron gun ⊖→
buncher
bunches of electron