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HOMEWORK 1

1 EVALUATE THE FOLLOWING INTEGRALS.

(a) $\int x^3 \cos(5x)\,dx$

	u	dv
$+$	x^3	$\cos(5x)$
$-$	$3x^2$	$\frac{1}{5}\sin(5x)$
$+$	$6x$	$\frac{-1}{25}\cos(5x)$
$-$	6	$\frac{-1}{125}\sin(5x)$
$+$	0	$\frac{1}{625}\cos(5x)$

$$= \boxed{\frac{x^3}{5}\sin(5x) + \frac{3x}{25}\cos(5x) - \frac{6x}{125}\sin(5x) - \frac{6}{625}\cos(5x) + C}$$

(b) $\int x^2 \sin(2x^3)\,dx$ $\qquad u = 2x^3$

$du = 6x^2\,dx$

$$= \frac{1}{6}\int \sin(u)\,du$$

$$= -\frac{1}{6}\cos(u) + C \quad = \boxed{-\frac{1}{6}\cos(2x^3) + C}$$

(c) $\int \frac{x^2}{x^2+1}\,dx$

$$\begin{array}{r|l} & 1 - \frac{1}{x^2+1} \\ \hline x^2+1 & x^2 + 0 \\ & -(x^2+1) \\ \hline & -1 \end{array}$$

$$= \int \left[1 - \frac{1}{x^2+1}\right] dx$$

$$= \boxed{x - \tan^{-1}(x) + C}$$

(d) $\int \frac{4-2x}{(x^2+1)(x-1)^2}\,dx = \int \frac{Ax+B}{x^2+1} + \frac{C}{(x-1)} + \frac{D}{(x-1)^2}\,dx$

$$(Ax+B)(x-1)^2 + C(x-1)(x^2+1) + D(x^2+1) = 4-2x$$

$$Ax^3 - 2Ax^2 + Ax + Bx^2 - 2Bx + B + Cx^3 - Cx^2 + Cx - C + Dx^2 + D = 4 - 2x$$

$$(A+C)x^3 + (-2A+B-C+D)x^2 + (A-2B+C)x + (B-C+D) = 4-2x$$

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$A+C=0$ $\qquad$ $A=2$

$-2A+B-C+D=0$ $\qquad$ $B=1$

$A-2B+C=-2$ $\qquad$ $C=-2$

$B-C+D=4$ $\qquad$ $D=1$

$$= \int \frac{2x+1}{x^2+1} + \frac{-2}{x-1} + \frac{1}{(x-1)^2}\,dx$$

$$= \int \frac{2x}{x^2+1} + \frac{1}{x^2+1} - \frac{2}{x-1} + \frac{1}{(x-1)^2}\,dx$$

$$= \boxed{\ln|x^2+1| + \arctan|x| - 2\ln|x-1| - \frac{1}{x-1} + C}$$

(e) $\int \frac{5x}{3x-1}\,dx$ $\qquad$ $u=3x-1 \Rightarrow x=\frac{u+1}{3}$

$du = 3\,dx$

$$= \frac{1}{3}\int \frac{5\left(\frac{u+1}{3}\right)}{u}\,du = \frac{5}{9}\int \frac{u+1}{u}\,du$$

$$= \frac{5}{9}\int\left(\frac{u}{u}+\frac{1}{u}\right)du = \frac{5}{9}\left[u+\ln|u|\right] + C$$

$$= \boxed{\frac{5}{9}\left[3x-1+\ln|3x-1|\right] + C}$$

$\boxed{2}$ ASSUMING THAT $s \in \mathbb{R}$, EVALUATE THE FOLLOWING IMPROPER INTEGRALS.

(a) $\int_0^\infty x^3 e^{\beta t} e^{-st}\,dt$ WHERE $\beta \in \mathbb{R}$ AND $s > \beta$

$$= \frac{x^3 e^{(\beta-s)t}}{\beta-s}\Bigg|_0^\infty = \lim_{t\to\infty} \frac{x^3 e^{(\beta-s)t}}{\beta-s} - \frac{x^3 e^{(\beta-s)(0)}}{\beta-s}$$

$$= 0 - \frac{x^3}{\beta-s} = \boxed{\frac{x^3}{\beta-s}}$$

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(b) $\int_0^\infty e^{-st}\cos(wt)\,dt$ WHERE $w \in \mathbb{R}$ AND $s > 0$

	u	dv
+	e^{-st}	$\cos(wt)$
−	$-se^{-st}$	$\frac{1}{w}\sin(wt)$
+	s^2e^{-st}	$\frac{-1}{w^2}\cos(wt)$

$$\Rightarrow \int_0^\infty e^{-st}\cos(wt)\,dt = \frac{e^{-st}}{w}\sin(wt) - \frac{se^{-st}}{w^2}\cos(wt) - \frac{s^2}{w^2}\int_0^\infty e^{-st}\cos(wt)\,dt$$

$$+\frac{s^2}{w^2}\int_0^\infty e^{-st}\cos(wt)\,dt \qquad\qquad + \frac{s^2}{w^2}\int_0^\infty e^{-st}\cos(wt)\,dt$$

$$\Rightarrow \left(1+\frac{s^2}{w^2}\right)\int_0^\infty e^{-st}\cos(wt)\,dt = \frac{e^{-st}}{w}\sin(wt) - \frac{se^{-st}}{w^2}\cos(wt)\Big|_0^\infty$$

$$\Rightarrow \int_0^\infty e^{-st}\cos(wt)\,dt = \frac{w^2}{w^2+s^2}\left[\frac{e^{-st}}{w}\sin(wt) - \frac{se^{-st}}{w^2}\cos(wt)\right]_0^\infty$$

$$= \lim_{t\to\infty}\left[\frac{w^2}{w^2+s^2}\left[\frac{e^{-st}}{w}\sin(wt) - \frac{se^{-st}}{w^2}\cos(wt)\right]\right] - \left[\frac{w^2}{w^2+s^2}\left(\frac{-s}{w^2}\right)\right]$$

$$= 0 + \frac{s}{w^2+s^2} = \boxed{\frac{s}{w^2+s^2}}$$

(c) $\int_0^\infty e^{-st}\sin(wt)\,dt$ WHERE $w \in \mathbb{R}$ AND $s > 0$

	u	dv
+	e^{-st}	$\sin(wt)$
−	$-se^{-st}$	$\frac{-1}{w}\cos(wt)$
+	s^2e^{-st}	$\frac{-1}{w^2}\sin(wt)$

$$\int_0^\infty e^{-st}\sin(wt)\,dt = \frac{-e^{-st}}{w}\cos(wt) - \frac{se^{-st}}{w^2}\sin(wt) - \frac{s^2}{w^2}\int_0^\infty e^{-st}\sin(wt)\,dt$$

$$+\frac{s^2}{w^2}\int_0^\infty e^{-st}\sin(wt)\,dt \qquad\qquad + \frac{s^2}{w^2}\int_0^\infty e^{-st}\sin(wt)\,dt$$

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$$\left(1+\frac{s^2}{w^2}\right)\int_0^{\infty} e^{-st}\sin(wt)\,dt = \left[\frac{-e^{-st}}{w}\cos(wt) - \frac{se^{-st}}{w^2}\sin(wt)\right]_0^{\infty}$$

$$= \frac{w^2}{w^2+s^2}\left[\frac{-e^{-st}}{w}\cos(wt) - \frac{se^{-st}}{w^2}\sin(wt)\right]_0^{\infty}$$

$$= \lim_{t\to\infty}\left[\frac{w^2}{w^2+s^2}\left(\frac{-e^{-st}}{w}\cos(wt) - \frac{se^{-st}}{w^2}\sin(wt)\right)\right] - \left[\frac{w^2}{w^2+s^2}\left(\frac{-1}{w}\right)\right]$$

$$= 0 + \frac{w}{w^2+s^2} = \boxed{\frac{w^2}{w^2+s^2}}$$

$\boxed{3}$ SOLVE THE FOLLOWING EQUATIONS FOR X.

(a) $\ln(x) - \ln(x-4) = -13$

$$\Rightarrow \ln\left(\frac{x}{x-4}\right) = -13$$

$$\Rightarrow e^{\ln\left(\frac{x}{x-4}\right)} = e^{-13}$$

$$\Rightarrow \frac{x}{x-4} = e^{-13}$$

$$\Rightarrow x = e^{-13}(x-4)$$

$$\Rightarrow x - xe^{-13} = -4e^{-13}$$

$$\Rightarrow \boxed{x = \frac{-4e^{-13}}{1-e^{-13}}}$$

(b) $\ln(x) + \ln(x-4) = -13$

$$\Rightarrow \ln(x(x-4)) = -13$$

$$\Rightarrow e^{\ln(x(x-4))} = e^{-13}$$

$$\Rightarrow x(x-4) = e^{-13}$$

$$\Rightarrow x^2 - 4x - e^{-13} = 0$$

$$\boxed{x = \frac{4 \pm \sqrt{16+4e^{-13}}}{2}}$$

Choose only positive Root

Note $x = \frac{4 - \sqrt{16+4e^{-13}}}{2} < 0$ and not in the domain of $\ln(x)$.

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(c) $e^{2(\ln(x)-\ln(x^2))}=1$

$\Rightarrow e^{2(\ln(\frac{x}{x^2}))}=1$

$\Rightarrow e^{\ln((\frac{x}{x^2})^2)}=1$

$\Rightarrow \frac{x^2}{x^4}=1 \Rightarrow x^2=1$

$\boxed{x=\pm 1}$ Again take only positive ~~R~~ x.

4 FIND ALL POINTS (x,y) WHICH SOLVE THE SIMULTANEOUS SYSTEMS.

(a) $4x-7y=1$ $\Rightarrow$ $24x-42y=6$ $\Rightarrow x=\frac{13}{45}$

$3x+6y=1$ $\quad +21x+42y=7$

$45x \quad =13$

BACK SUBSTITUTING:

$4(\frac{13}{45})-7y=1 \Rightarrow y=\frac{1}{45}$ $\boxed{(\frac{13}{45},\frac{1}{45})}$

(b) $2x-\frac{2x^2}{3}-xy=0$

$4xy-16y=0$

IF $x=y=0$ THEN THE EQUATIONS ARE TRUE AND $(0,0)$ IS A SOLUTION

IF $x\neq 0, y\neq 0$:

$\Rightarrow$ $2-\frac{2}{3}x-y=0$

$4x-16=0$ $\Rightarrow 4x=16 \Rightarrow x=4$

BACK SUBSTITUTING:

$2-\frac{2}{3}(4)-y=0 \Rightarrow y=-\frac{2}{3}$

THE SOLUTIONS ARE $\boxed{(0,0),(4-\frac{2}{3})}$

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(c) $yx^2+y^3-y=0$

$x-x^3-xy^2=0$

$\Rightarrow x^2+y^2-1=0$

$1-x^2-y^2=0$

$\Rightarrow x^2+y^2=1$

$x^2+y^2=1$

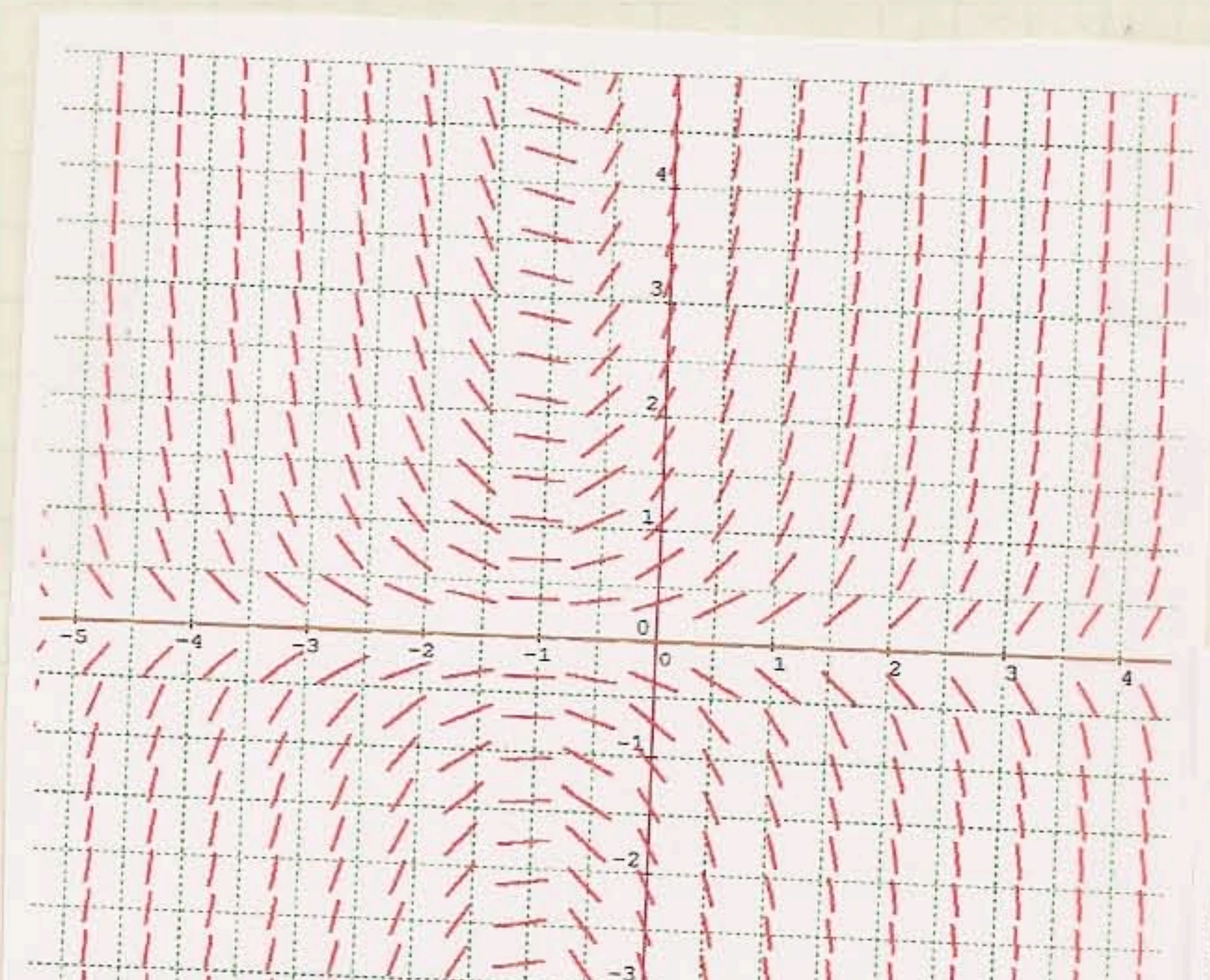
ALL POINTS ON THE CIRCLE $x^2+y^2=1$ ARE SOLUTIONS

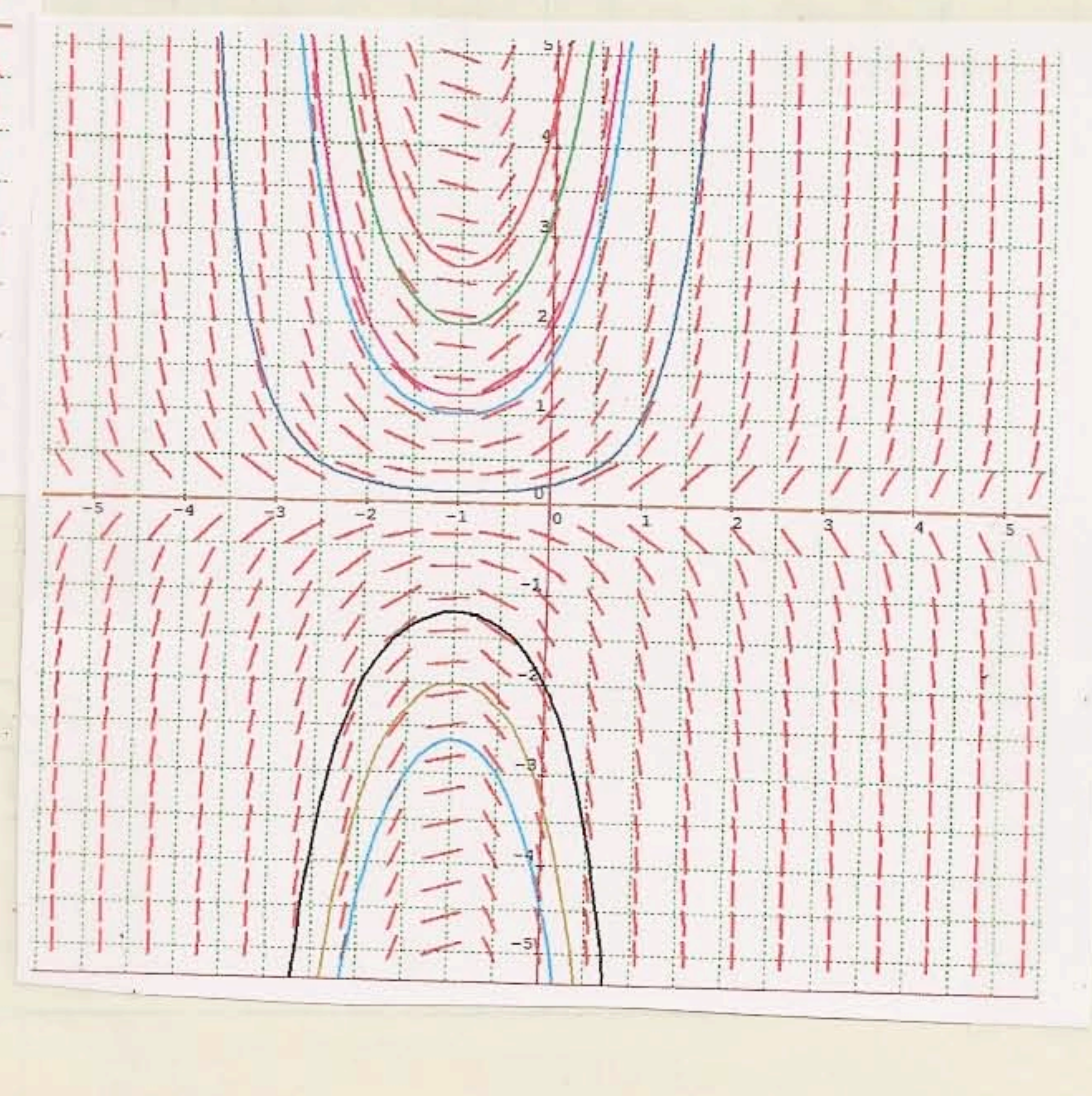
$$\boxed{\left(x, \pm\sqrt{1-x^2}\right)}$$

$\boxed{5}$ CONSIDER THE O.D.E. $\frac{dy}{dt}=(1+t)y$

(a) PLOT THE VECTOR FIELD

(b) AND THE SOLUTION CURVES PASSING THROUGH THE POINTS $(-1,2),(-1,-2),(-.5,3),(-1,0),(-.5,-3),(-2,2),(-2,-2),(1,1),(-1,1)$





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(c) VERIFY THAT $y(t) = Ce^{(\frac{1}{2}t^2+t)}$ SATISFIES THE O.D.E REGARDLESS OF THE CHOICE OF C

$$y(t) = Ce^{(\frac{1}{2}t^2+t)}$$

$$\frac{dy}{dt} = Ce^{(\frac{1}{2}t^2+t)}(t+1)$$

SUBSTITUTING $y(t)$ AND $\frac{dy}{dt}$ INTO THE O.D.E., WE GET

$$\frac{dy}{dt} = (1+t)y$$

$$\Rightarrow Ce^{(\frac{1}{2}t^2+t)}(1+t) = (1+t)Ce^{(\frac{1}{2}t^2+t)}$$

Problem 5:

a) $y' = 0 \Rightarrow y(t) = C, \ C \in \mathbb{R}$

b) $y' = y \Rightarrow y(t) = Ce^{t}, \ C \in \mathbb{R}$ Note $C = 0 \Rightarrow y = 0$ which is a soln

$\Downarrow$

c) $y'' = y \Rightarrow y(t) = Ce^{t}, \ C \in \mathbb{R}$, Again same is true
$y(t) = Ce^{-t}$

$\Downarrow$

d) $y'' = y \Rightarrow y(t) = C\cos(t), \ C \in \mathbb{R}$, " "
$y(t) = C\sin(t)$