Jan 18 lec covers section 1-4 and 2-4 along with material on conservation
laws which is not in the book (references are given below).

Review for the Monday 1/2 exam (30 minutes):

-one problem on deriving areas or volumes of circles, cylinders, or spheres

-two problems on setting up the integral for the electric field from either a
line or surface charge distribution.

You will only have to justify the set-up of the integral with limits correctly in these problems.

InkSurvey muddy points:

- flux thru strm surfaces: $\int \vec{E} \cdot da \propto \int \frac{1}{r^2} da \cos 4$
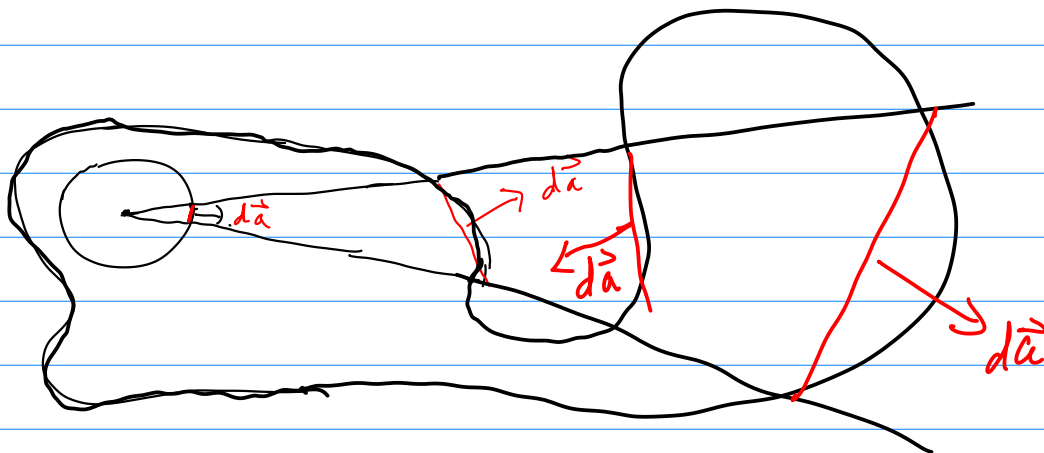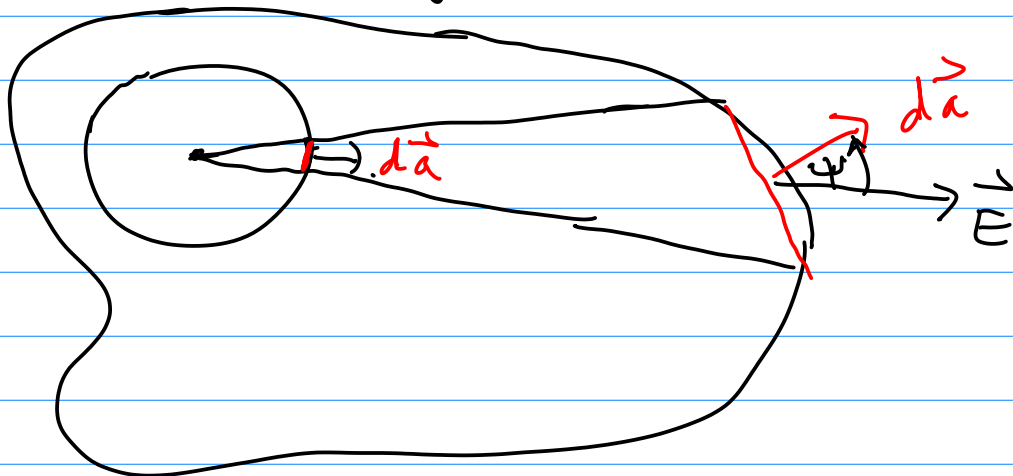
What happens with a charge offset from the center of the first sphere?

Solid angle: http://en.wikipedia.org/wiki/Solid_angle $\int d\Omega = \int \sin\vartheta \, d\vartheta \, d\ell = 4\pi$

$$-\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0 \uparrow}$$

$\underbrace{\qquad}$ neg flux   neg charge

How do you get flux from differential form?
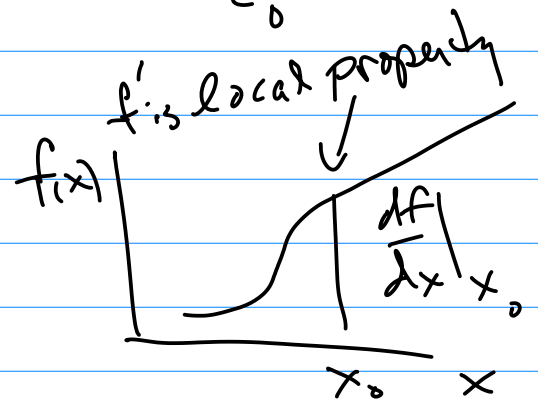
$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$

$$\int \vec{\nabla} \cdot \vec{E} \, d\tau = \frac{1}{\epsilon_0} \int \rho \, d\tau$$

div. theorem $\Downarrow$ $\uparrow$ volume $dx\,dy\,dz$

f is local property

$f(x)$

$$\left. \frac{df}{dx} \right|_{x_0}$$

x_0 x

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$$

Fundamental theorem of calculus

$$\int_a^b \frac{df}{dx} dx = f(b) - f(a)$$

sum changes in f to get total change at bndry

Integal of derivative over interval is the value of function at end points (bndries)

Fundamental theorem of diverences

$$\int \vec{\nabla} \cdot \vec{F} \, d\tau = \oint \vec{F} \cdot d\vec{a}$$

Integral of derivative (divergence) over region (volume) is value of function at bndry (surface)

- Analogy questions like $\vec{\nabla} \cdot \vec{B} = ?$ $\vec{\nabla} \times \vec{E} = ?$

$$\oint \vec{B} \cdot d\vec{a} =$$
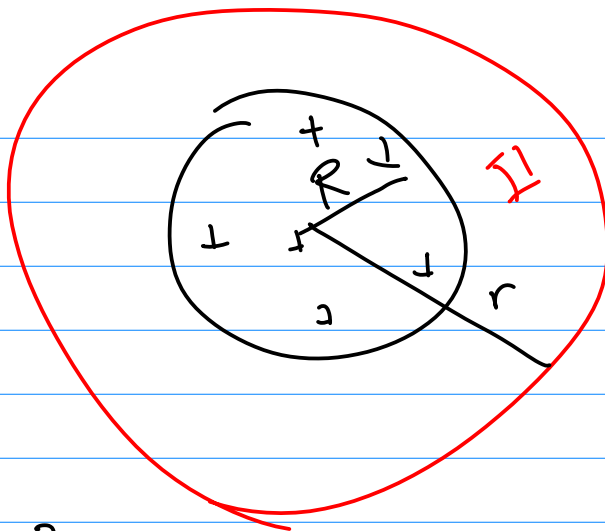
-Advantages of differential form: Coulombs law becomes a partial differential equation.

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$ Use numerical method to iterate solns
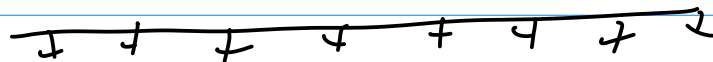
Examples of Gauss's law

uniformly charged sphere

$$\oint \vec{E} \cdot d\vec{a} = E \, 4\pi R^2$$

$$\frac{Q_{encl}}{\epsilon_0} = \frac{\sigma \, 4\pi r^2}{\epsilon_0}$$
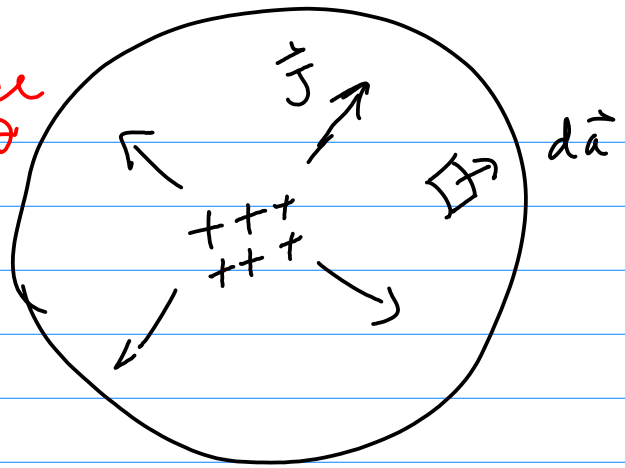
Close to the surface

Infinite plane of charge

Superposition principle

Conservation of charge continued.

by this surface

$$\oint \vec{J} \cdot d\vec{a} = -\frac{dQ_{enclosed}}{dt}$$

$$Q_{enclosed} = \int \rho \, dvolume$$

$$\overset{\shortparallel}{dxdydz \text{ or } r^2 \sin\theta \, d\theta \, d\phi \, dr}$$

$$-\frac{dQ_{enclosed}}{dt} = \int -\frac{\partial\rho}{\partial t} \, dxdydz$$

$$\overset{//}{\oint \vec{J} \cdot d\vec{a}} = \int \vec{\nabla} \cdot \vec{J} \, dx \, dy \, dz \quad \text{Divergence theorem}$$

Differential form of conservation of charge is

$$\boxed{\vec{\nabla} \cdot \vec{J} = -\frac{\partial\rho}{\partial t}}$$

Also called the CONTINUITY EQUATION

http://en.wikipedia.org/wiki/Continuity_equation

this link gives examples for 4-current in special relativity, fluid dynamics, thermodynamics, quantum mechanics, general relativity, quantum chromodynamics

traffic flow continuity equation

Examples of vector calculus in fluids

http://en.wikipedia.org/wiki/Navier%E2%80%93Stokes_equations