MATH 235 - Differential Equations w/ Honors January 14, 2008

Homework 2, Spring 2008 **Due**: January 21, 2008

1. Solutions to ordinary differential equations.

   (a) Show that $y(t) = e^{2t} + ce^{t}$, where $c \in \mathbb{R}$, is a solution to $\dfrac{dy}{dt} - y = e^{2t}$.

   (b) Show that $x^2 + y^2 = cx$, where $c \in \mathbb{R}$, is a solution to $2xy\dfrac{dy}{dx} = y^2 - x^2$.

   (c) Show that $y(t) = c_1\sinh(t) + c_2\cosh(t)$, where $c_1, c_2 \in \mathbb{R}$, is a solution to $y'' - y = 0$.

   (d) Show that $y(t) = c_1 \sin(t) + c_2 \cos(t)$, where $c_1, c_2 \in \mathbb{R}$, is a solution to $y'' + y = 0$.

   (e) Show that $x(t) = Ae^{-k_1 t}$ and $y(t) = \dfrac{k_1 A}{k_1 - k_2}e^{-k_2 t} + \dfrac{k_1 A}{k_2 - k_1}e^{-k_1 t}$ are solutions to the system of differential equations.

$$\frac{dx}{dt} = -k_1 x, \quad x(0) = A \tag{1}$$

$$\frac{dy}{dt} = k_1 x - k_2 y, \quad y(0) = 0 \tag{2}$$

   Hint: For problem 1c recall that $\sinh(t) = \dfrac{e^t - e^{-t}}{2}$, $\cosh(t) = \dfrac{e^t + e^{-t}}{2}$.

2. Solve the following problems via separation of variables. When appropriate solve for the integrating constant C using the initial value which is given.

   (a) $\dfrac{dy}{dt} = 1 + \dfrac{1}{y^2}$.

   (b) $(y')^2 - xy' + y = 0$.

   (c) $\dfrac{dy}{dt} = (y^2 + 1)t$, $y(0) = 1$.

   (d) $\dfrac{dy}{dt} = \dfrac{ye^t}{1 + y^2}$.

   Hint: For (b) consider completing the square and using the variable substitution $z = -(y - t^2/4)$.

3. Section 1.3 of the text, problems 8, 10, 15.

4. Consider the polynomial $p(y) = -y^3 - 2y + 2$.

   (a) Using HPGSolver sketch the slope field for $\dfrac{dy}{dt} = p(y)$.

   (b) Using HPGSolver, sketch the graphs of some of the solutions using the slope field.

   (c) Describe the relationship between the roots of $p(y)$ and the solutions of the differential equation.

   (d) Using Euler's method, approximate the real root(s) of $p(y)$ to three decimal places.

5. The following nonlinear system has been proposed as a model for a predator-prey system of two particular species of microorganisms.

$$\frac{dx}{dt} = ax - by\sqrt{x} \tag{3}$$

$$\frac{dy}{dt} = cy\sqrt{x}, \tag{4}$$

   where $a, b, c \in \mathbb{R}^+$. In this case the variables x and y are dependent variables and appear in both ODE's and thus the ODE's are said to be coupled.

   (a) Which variable, x or y, represents the predator population? Which variable represents the prey population? Justify your choices.

   (b) What happens to the predator population if the prey is extinct? Justify your conclusion.