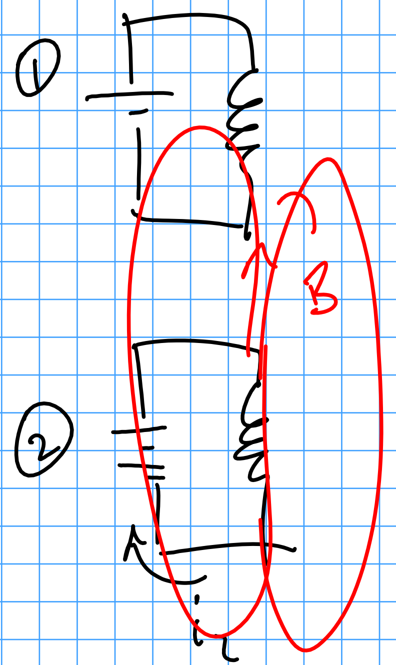
Review



$$\boxed{\Phi_1 = N_1 \Phi_0 = M_{12}\, i_2}$$

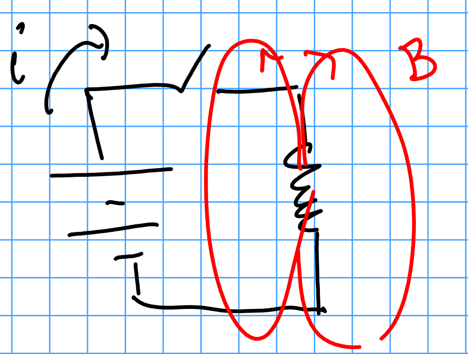
$$\mathcal{E}mf_1 = -\frac{d\Phi_1}{dt} = -M_{12}\frac{di_2}{dt}$$

$$\Phi_2 = N_2 \Phi_0 = M_{21}\, i_1$$

$$\mathcal{E}mf_2 = -\frac{d\Phi_2}{dt} = -M_{21}\frac{di_1}{dt}$$

$$M_{12} = M_{21}$$

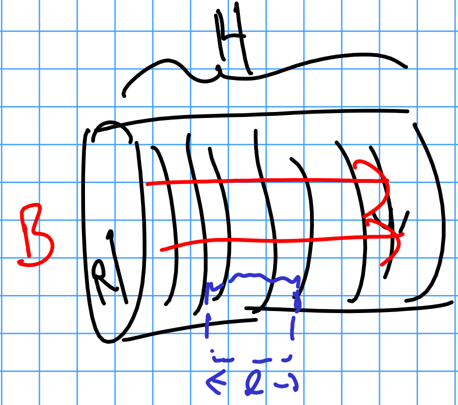
Self inductance



$$\Phi \propto i$$

$$\Phi_{tot} = \boxed{N \Phi_0 \equiv L\, i}$$

$$\mathcal{E}mf = -\frac{d\Phi}{dt} = -L\frac{di}{dt}$$



$n \; \frac{\text{turns}}{\text{length}}$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} \qquad B\cancel{l} = \mu_0 n \cancel{l} I \qquad B = \mu_0 n I$$

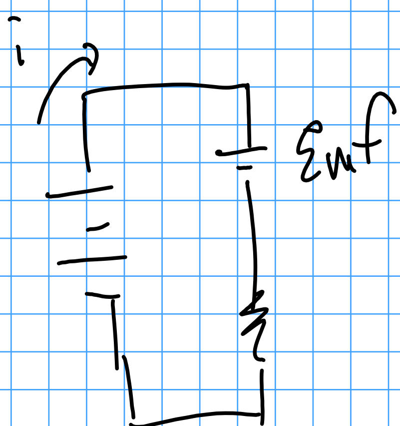
$$\Phi_0 = \int \vec{B}\cdot d\vec{a} = B \underbrace{\int da}_{\pi R^2}$$

$$\Phi_0 = \mu_0 n I \pi R^2$$

$$\Phi_{total} = N \mu_0 n \cancel{I} \pi R^2 = L \cancel{I}$$

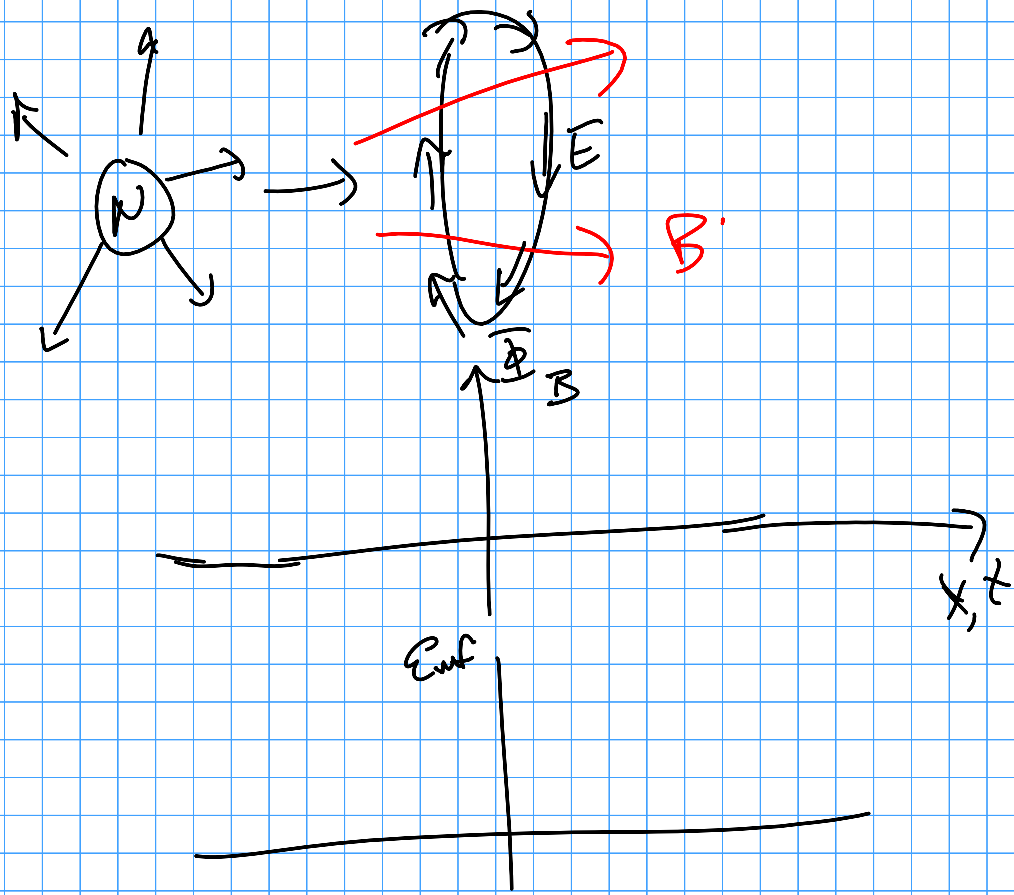
$$N = nH$$

$$\frac{N}{H} = n$$



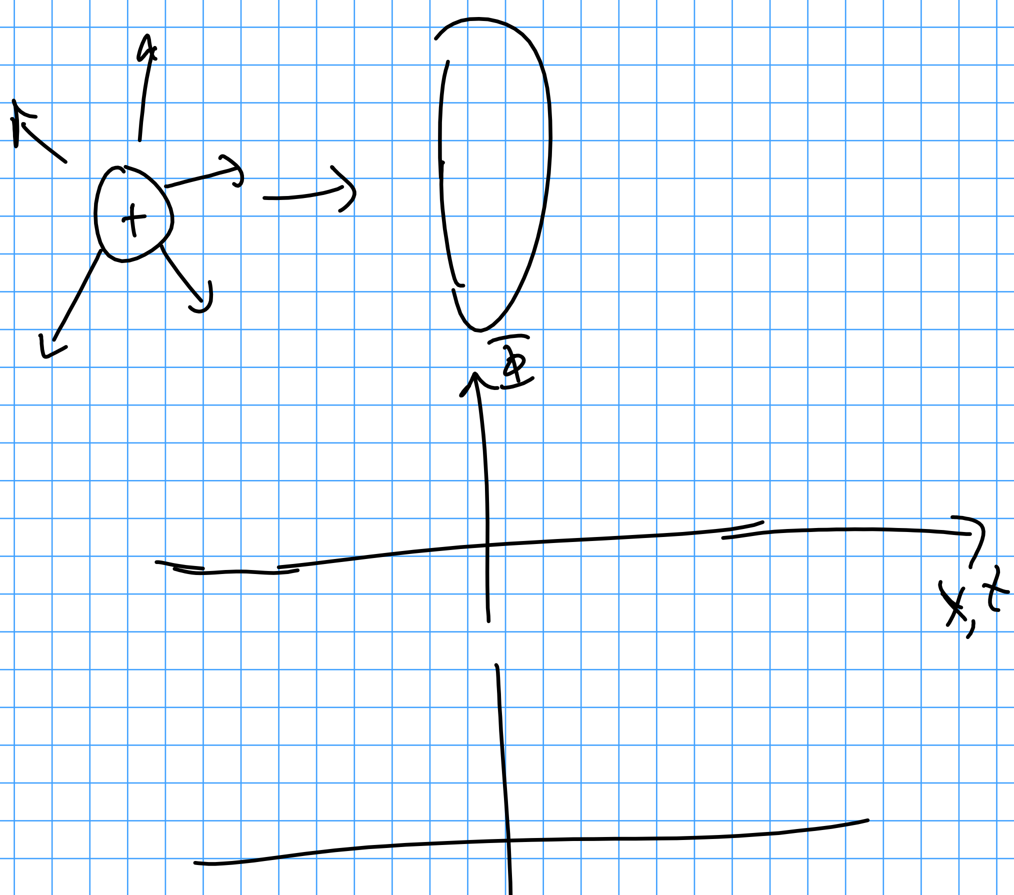
Drop problem #26 for this Wednesday

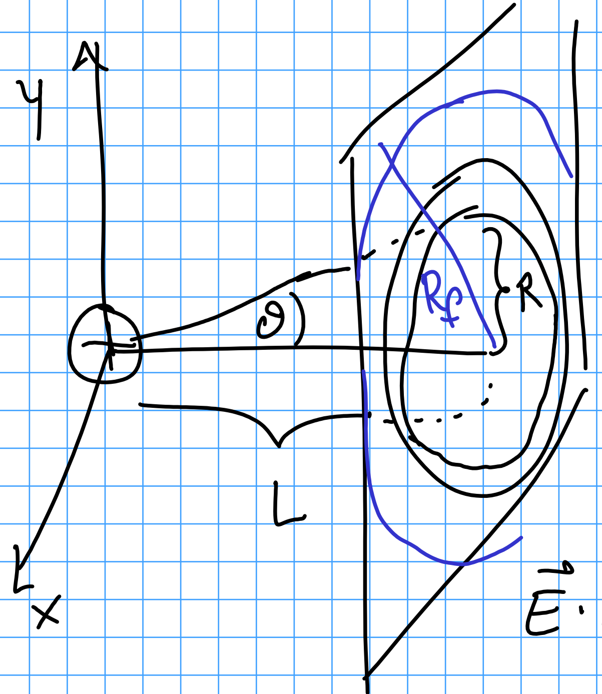
Faraday's law



$$\text{Emf} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$$





$$\vec{E} = \frac{kq}{r^2}\hat{r} \qquad |\vec{r}| = \sqrt{R^2+L^2}$$

$$\Phi_E = \int \vec{E}\cdot d\vec{a} \qquad d\vec{a} = 2\pi R\, dR\, \hat{z}$$

$$\hat{r} = \hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$

$$\vec{E}\cdot d\vec{a} = \frac{q}{4\pi\epsilon_0}\frac{1}{R^2+L^2}\,2\pi R\,\underbrace{\hat{r}\cdot\hat{z}}_{\cos\theta = \frac{L}{\sqrt{R^2+L^2}}}$$

$$\Phi_E = \int_0^{R_0} \frac{q}{4\pi\epsilon_0}\frac{2\pi R\,dR}{(R^2+L^2)}\frac{L}{\sqrt{R^2+L^2}} = \frac{qL}{2\epsilon_0}\int_0^{R}\frac{R\,dR}{(R^2+L^2)^{3/2}}$$

let $\tan\theta = \frac{R}{L}$ $\quad dR = L\sec^2\theta\, d\theta = \frac{L}{\cos^2\theta}d\theta$

$$\cos\theta = \frac{L}{\sqrt{L^2+R^2}} \Rightarrow \sqrt{L^2+R^2} = \frac{L}{\cos\theta}$$

$$\sin\theta = \frac{R}{\sqrt{L^2+R^2}}$$

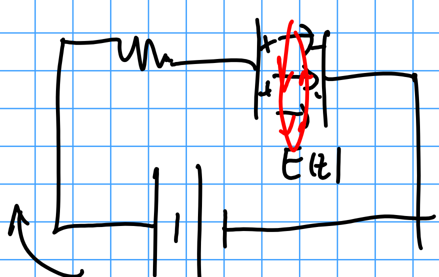
$$\Phi_E = \frac{qL}{2\epsilon_0}\int_0^{\theta_f} \underbrace{\frac{\cos^2\theta}{L^2}}_{\frac{1}{L^2+R^2}}\,\underbrace{\sin\theta}_{\frac{R}{\sqrt{L^2+R^2}}}\,\underbrace{\frac{L}{\cos^2\theta}d\theta}_{dR} = \frac{q}{2\epsilon_0}\int_0^{\theta_f}\sin\theta\, d\theta$$

$$\boxed{\Phi_E = -\frac{q}{2\epsilon_0}\left(\cos\theta_f - 1\right)}$$

$$\cos\theta_f = \frac{L}{\sqrt{L^2+R_0^2}}$$

as $L \to \pm\infty$ $\quad \theta_f \to 0 \quad \cos\theta_f \to 1 \quad \Phi \to 0$

as $L \to 0$ $\quad \theta_f \to \pi/2 \quad \cos\theta_f \to 0 \quad \Phi \to \frac{q}{2\epsilon_0}$



$\vec\nabla\cdot\vec E = \rho/\epsilon_0 \qquad \vec\nabla\cdot\vec B = 0$

$\vec\nabla\times\vec B = \mu_0\vec J \qquad \vec\nabla\times\vec E = -\frac{\partial\vec B}{\partial t}$

i(t) not magnetostatics

$\vec\nabla\cdot\vec\nabla\times\vec C = \emptyset$

$\uparrow$ vector function

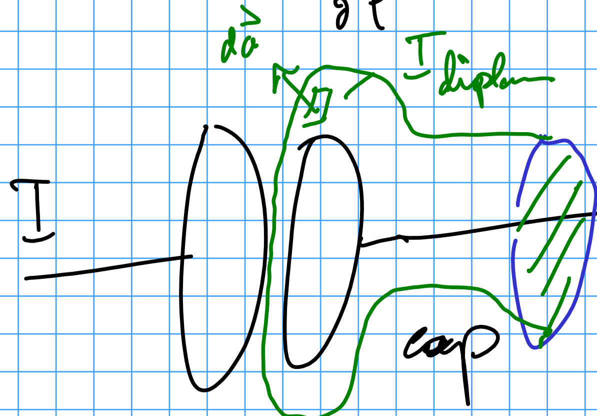
$\vec\nabla\cdot\vec\nabla\times\vec E = -\frac{\partial}{\partial t}\underbrace{\vec\nabla\cdot\vec B}_{0} = \emptyset$ ok

$\vec\nabla\cdot\left(\vec\nabla\times\vec B\right) = \mu_0\vec\nabla\cdot\vec J = -\frac{\partial\rho}{\partial t} \neq 0$ Problem with $\vec\nabla\times\vec B = \mu_0\vec J + ?$

$\vec\nabla\cdot\vec E = \rho/\epsilon_0$

$$\vec\nabla\times\vec B = \underset{\text{physical current}}{\mu_0 \vec J} + \epsilon_0\mu_0 \underbrace{\frac{\partial \vec E}{\partial t}}_{\text{displacement current}}$$

$$\vec\nabla\cdot(\vec\nabla\times\vec B) = \mu_0 \underbrace{\vec\nabla\cdot\vec J}_{-\frac{\partial\rho}{\partial t}} + \epsilon_0\mu_0\frac{\partial}{\partial t}\underbrace{\vec\nabla\cdot\vec E}_{\rho/\epsilon_0} = 0$$

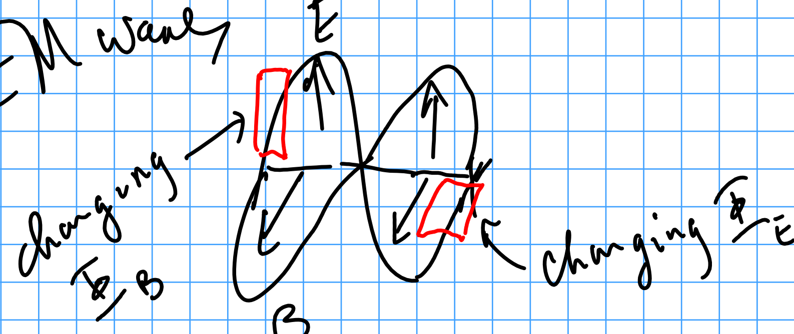


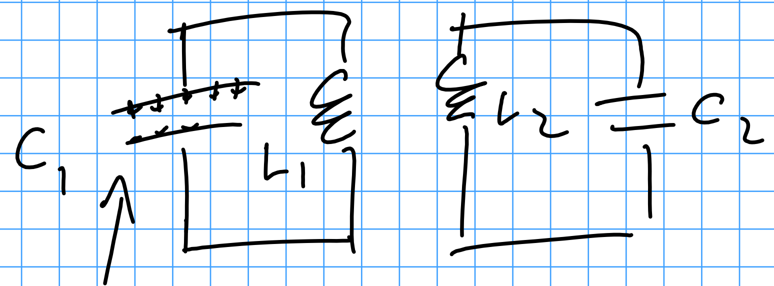
$$\oint \vec B\cdot d\vec l = \int \vec\nabla\times\vec B\cdot d\vec a$$
$$= \mu_0 I_{encl}$$

$$J_d = \mu_0\epsilon_0 \frac{\partial \vec E}{\partial t} \qquad \vec E = \frac{\sigma}{\epsilon_0} \qquad \sigma = \frac{Q}{A}$$

$$= \mu_0\epsilon_0\frac{1}{\epsilon_0}\frac{\partial\sigma}{\partial t} = \frac{\mu_0}{A}\frac{\partial Q}{\partial t} = \frac{\mu_0}{A}I(t)$$

EM wave





$V_{cap} = \frac{Q}{C}$

Kirchoff's law

$Q_{initial}$ $\quad M_{12} = M_{21} = M$

$i_1(t),\ i_2(t)$

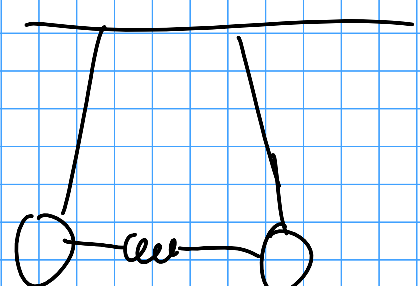
$$\frac{Q_1(t)}{C_1} - L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0$$

$$\frac{Q_2(t)}{C_2} - L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = 0$$

2 Coupled ODE's: Laplace transform

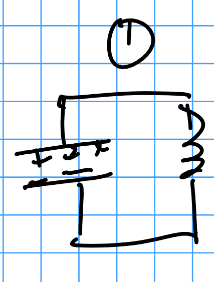
fourier transform

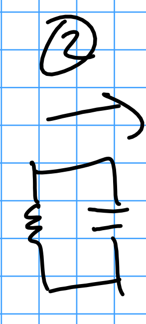
guess $Q(t) = A\sin(\omega_1 t + \varphi_1)$



Some math

Perturbation Theory:





$i_{1,0}$ given by $\frac{Q_1}{C_1} - L_1\frac{di_{1,0}}{dt} = 0$ $\quad i_{2,0} \Rightarrow \frac{Q_2}{C_2} - L_2\frac{di_{2,0}}{dt} = 0$

$i_{1,1}$ given by $\frac{Q_1}{C_1} - L_1\frac{di_{1,1}}{dt} - M\frac{di_{2,0}}{dt} = 0$ $\quad i_{2,1}$ $\frac{Q_2}{C_2} - L_2\frac{di_{2,1}}{dt} - M\underbrace{\frac{di_{1,0}}{dt}}_{=0}$

$$i = i_{1,0} + i_{1,1} + i_{1,2} + \cdots$$