

EXAMPLE**2.1**

Given point $P(-2, 6, 3)$ and vector $\mathbf{A} = y\mathbf{a}_x + (x+z)\mathbf{a}_y$, express P and \mathbf{A} in cylindrical and spherical coordinates. Evaluate \mathbf{A} at P in the Cartesian, cylindrical, and spherical systems.

SOLUTION

At point P : $x = -2, y = 6, z = 3$. Hence,

$$\rho = \sqrt{x^2 + y^2} = \sqrt{4 + 36} = 6.32$$

$$\phi = \tan^{-1} \frac{y}{x} = \tan^{-1} \frac{6}{-2} = 108.43^\circ$$

$$z = 3$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{4 + 36 + 9} = 7$$

$$\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z} = \tan^{-1} \frac{\sqrt{40}}{3} = 64.62^\circ$$

Thus, \vec{P}

$$P(-2, 6, 3) = P(6.32, 108.43^\circ, 3) = P(7, 64.62^\circ, 108.43^\circ)$$

In the Cartesian system, \mathbf{A} at P is

$$\mathbf{A} = 6\mathbf{a}_x + \mathbf{a}_y$$

For vector \mathbf{A} , $A_x = y, A_y = x + z, A_z = 0$. Hence, in the cylindrical system

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ x + z \\ 0 \end{bmatrix}$$

or

$$A_\rho = y \cos \phi + (x + z) \sin \phi$$

$$A_\phi = -y \sin \phi + (x + z) \cos \phi$$

$$A_z = 0$$

But $x = \rho \cos \phi, y = \rho \sin \phi$, and substituting these yields

$$\begin{aligned} \mathbf{A} = (A_\rho, A_\phi, A_z) &= [\rho \cos \phi \sin \phi + (\rho \cos \phi + z) \sin \phi] \mathbf{a}_\rho \\ &+ [-\rho \sin^2 \phi + (\rho \cos \phi + z) \cos \phi] \mathbf{a}_\phi \end{aligned}$$

At P

$$\rho = \sqrt{40}, \quad \tan \phi = \frac{6}{-2}$$