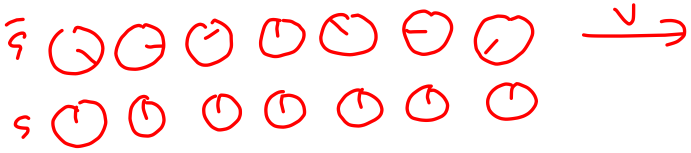

If we're looking for how time flows for an object in $\bar{S}$ $\Rightarrow \bar{x} = \text{const.}$

$\bar{x} = \gamma(x - vt)$

$\bar{y} = y$

$\bar{z} = z$

$\bar{t} = \gamma(t - \frac{v}{c^2}x)$

$\Delta t = \gamma \Delta \bar{t}$

$\Delta \bar{t} = \frac{1}{\gamma}\Delta t$

$x = \gamma(\bar{x} + v\bar{t})$

$y = \bar{y}$

$z = \bar{z}$

$t = \gamma(\bar{t} + \frac{v}{c^2}\bar{x})$

$p_\mu p^\mu = -\frac{E^2}{c^2} + p^2 = -m^2c^2$ } $E^2 - (pc)^2 = m^2c^4$

$E_1 + E_2$; $\bar{E}_1$; $\bar{E}_2$; $\bar{p}_1$; $\{\bar{p}_2 = \phi\}$

$(2E)^2 - \phi = (\bar{E} + mc^2) - (\bar{p}c)^2 = m^{*2}c^4$ } Both particles

$-c^2 p_\mu p^\mu$ in S ; $-c^2 p_\mu p^\mu$ in $\bar{S}$

$\bar{E}^2 - (\bar{p}c)^2 = E^2 - (pc)^2 = +m^2c^4$ } Particle 1.

Problem 10.3: $\int \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2}\right) dV = \int 4\pi\delta^3(\vec{r})\, dV$

$\oint \frac{\hat{r}}{r^2} \cdot d\vec{a} = 4\pi$

$p_\mu p^\mu =$

$p^\mu = m\eta^\mu$

$\eta^\mu = \begin{pmatrix} \gamma c \\ \gamma \vec{u} \end{pmatrix}$

$\bar{\eta}^\mu = \Lambda^\mu_\nu \eta^\mu$

$\eta^\mu = \begin{pmatrix} \gamma c \\ \gamma u_x \\ \gamma u_y \\ \gamma u_z \end{pmatrix}$

$\eta^0 = \dfrac{dx^0}{d\tau} = c\dfrac{dt}{d\tau} = \gamma c$

$d\tau = \frac{1}{\gamma} dt$

$\eta^1 = \dfrac{dx^1}{d\tau} = \dfrac{dx}{d\tau} = \gamma \dfrac{dx}{dt} = \gamma u_x$

Maxwell's eqns

Conservation Laws:

momentum

$$\rho\vec{E}+\vec{J}\times\vec{B} = \vec{\nabla}\cdot\overleftrightarrow{T} - \epsilon_0\mu_0\frac{\partial\vec{S}}{\partial t}$$

$$\int \rho\vec{E}+\vec{J}\times\vec{B}\,dV = \oint \overleftrightarrow{T}\cdot d\vec{a} - \epsilon_0\mu_0\frac{d}{dt}\int\vec{S}\,dV$$

Force on Vol. — flow of momentum into vol. — Time der. of mom. in Vol.

momentum density:

$$\vec{\wp} = \epsilon_0\mu_0\vec{S}$$

charge

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\oint\vec{J}\cdot d\vec{a} = -\frac{d}{dt}\int\rho\,dV = -\frac{dq}{dt}$$

Energy:

$$\vec{E}\cdot\vec{J} = -\frac{\partial}{\partial t}\left(\frac{1}{2}\left(\epsilon_0E^2+\frac{1}{\mu_0}B^2\right)\right) - \vec{\nabla}\cdot\left(\frac{1}{\mu_0}\vec{E}\times\vec{B}\right)$$

u_{em}: e+m energy dens. — $\vec{S}$: energy flux density.

$$\int\vec{E}\cdot\vec{J}\,dV = -\frac{\partial}{\partial t}\int u_{em}\,dV - \oint\vec{S}\cdot d\vec{a}$$

Mech power taking energy out of fields. — time der. of energy in vol. — Flow of energy into vol per time

Ang. momentum:

$$\vec{\ell}_{em} = \vec{r}\times\vec{\wp} = \epsilon_0\left[\vec{r}\times(\vec{E}\times\vec{B})\right]$$

Waves:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{u} = \phi$$

Potentials:

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

$$V' = V - \frac{\partial \lambda}{\partial t}$$

$$\vec{A}' = \vec{A} + \vec{\nabla} \lambda$$

} for any λ and it don't change a thing.

Lorentz Gauge

$$\Box^2 V = -\frac{\rho}{\epsilon_0} \qquad \Box^2 \vec{A} = -\mu_0 \vec{J}$$

Special Relativity:

Very cool.

Time dialation: $\Delta\bar{t} = \frac{1}{\gamma}\Delta t$

Length contraction: $\Delta\bar{x} = \gamma\Delta x$

Lorentz Transforms: $\bar{x} = \gamma(x - vt)$

$\bar{y} = y$

$\bar{z} = z$

$\bar{t} = \gamma(t - \frac{v}{c^2}x)$

4-vectors: $x^\mu = (ct, x, y, z)$

$$\bar{x}^\mu = \frac{\partial\bar{x}^\mu}{\partial x^\nu}x^\nu = \Lambda^\mu_\nu x^\nu$$

$$\bar{x}_\mu = \frac{\partial x^\mu}{\partial\bar{x}^\nu}x^\nu$$

$$\eta^\mu = \frac{dx^\mu}{d\tau} \qquad d\tau = \frac{1}{\gamma}dt = \text{proper time}$$

$$p^\mu = m\eta^\mu = \begin{pmatrix} \gamma mc \\ \gamma m\vec{u} \end{pmatrix} = \begin{pmatrix} E/c \\ \gamma m\vec{u} \end{pmatrix}$$

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(\gamma m\vec{u})$$

$$K^\mu = \frac{dp^\mu}{d\tau}$$

$\bar{E}_x = E_x$ $\quad \bar{E}_y = \gamma(E_y - vB_z)$ $\quad \bar{E}_z = \gamma(E_z + vB_y)$

$\bar{B}_x = B_x$ $\quad \bar{B}_y = \gamma(B_y + \frac{v}{c^2}E_z)$ $\quad \bar{B}_z = \gamma(B_z - \frac{v}{c^2}E_y)$

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ & 0 & B_z & -B_y \\ -\text{That} & & 0 & B_x \\ & & & 0 \end{pmatrix} \quad G^{\mu\nu} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ & 0 & -E_z/c & E_y/c \\ -\text{Top} & & 0 & -E_x/c \\ & & & 0 \end{pmatrix}$$

$J^\mu = (\rho c, \vec{J})$

Cont. Equn: $\frac{\partial J^\mu}{\partial x^\mu} = \varnothing$

Max's Equns.

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = \mu_0 J^\mu \; ; \quad \frac{\partial G^{\mu\nu}}{\partial x^\nu} = \varnothing$$

Minkowski Force

$$K^\mu = q\eta_\nu F^{\mu\nu}$$

Potentials

$$A^\mu = (\frac{V}{c}, \vec{A}) \Rightarrow F^{\mu\nu} = \frac{\partial A^\nu}{\partial x_\mu} - \frac{\partial A^\mu}{\partial x_\nu}$$

$$\frac{\partial^2 A^\mu}{\partial x_\mu \partial x^\mu} = \mu_0 J^\mu = \Box^2 A^\mu = \mu_0 J^\mu$$