EXAM III next Wednesday

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 \qquad \vec{\nabla}\times\vec{E} = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

$$A_x(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{J_x(\vec{r}\,')\,d\tau'}{|\vec{r}-\vec{r}\,'|}$$

$$B = \frac{\mu_0}{4\pi}\int\frac{\vec{J}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau$$

$$\oint\vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$\begin{tikzcd}
 & \vec{J} \arrow[dl] \arrow[dr] & \\
\vec{A} \arrow[ur, bend left] \arrow[rr, "\vec{B}=\vec{\nabla}\times\vec{A}"'] & & \vec{B} \arrow[ul, "\vec{\nabla}\times\vec{B}=\mu_0\vec{J}"']
\end{tikzcd}$$

$$\boxed{\nabla^2\vec{A} = -\mu_0\vec{J}}$$

$$\vec{E} = -\vec{\nabla}V$$

Guess $\vec{B} = -\vec{\nabla}G$ then $\underbrace{\vec{\nabla}\times\vec{B}}_{=\mu_0\vec{J}} = \vec{\nabla}\times\vec{\nabla}G$

$$0 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} & \frac{\partial G}{\partial z} \end{vmatrix} = \hat{x}\left(\frac{\partial}{\partial y}\frac{\partial G}{\partial z} - \frac{\partial}{\partial z}\frac{\partial G}{\partial y}\right) + \cdots$$

Note $\vec{\nabla}\cdot\vec{\nabla}\times\vec{A} = 0$ for all $\vec{A}$

But $\vec{\nabla}\cdot\vec{B} = 0$ always

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

Scalar potential V was not unique up to a constant.

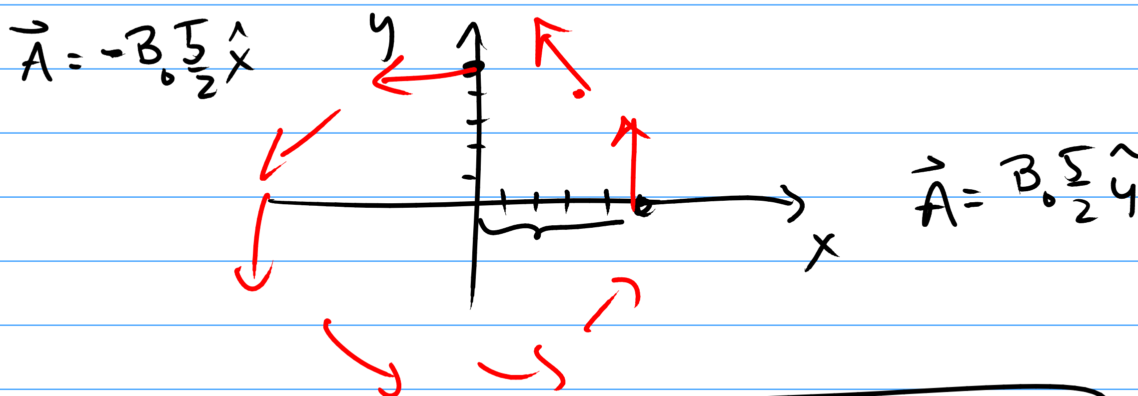
$$\vec{E} = -\vec{\nabla}(V + \text{const}) = -\vec{\nabla} V$$

similarly $\vec{A}$ is not unique

$$\vec{\nabla} \times \vec{A} \to \vec{\nabla} \times (\vec{A} + \vec{\nabla} \psi) = \vec{\nabla} \times \vec{A} + \underbrace{\vec{\nabla} \times \vec{\nabla} \psi}_{\text{zero for all } \psi}$$

INKSURVEY:

$$\vec{A} = -B_0 \frac{y}{2} \hat{x} + B_0 \frac{x}{2} \hat{y} + 0 \hat{z}$$



$$\vec{B} = \vec{\nabla} \times \vec{A} = B_0 \hat{z} \leftarrow \boxed{\vec{A} = -B_0 y \hat{x}}$$

Need $\vec{\nabla} \times \vec{A}$ & $\vec{\nabla} \cdot \vec{A}$ to specify $\vec{A}$

Since

$$\vec{\nabla} \cdot \vec{A} \to \vec{\nabla} \cdot (A + \vec{\nabla} \psi) = \vec{\nabla} \cdot \vec{A} + \nabla^2 \psi$$

we can make $\vec{\nabla} \cdot \vec{A}$ anything by a choice of ψ

(Coulomb Gauge)

For magnetostatics chose ψ such that $\boxed{\vec{\nabla} \cdot \vec{A} = 0}$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$
$$\parallel$$
$$\vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{A} = \mu_0\vec{J}$$

vector identity

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$
$$\parallel$$
$$-\vec{\nabla}V$$

$$\nabla^2 V = -\rho/\epsilon_0$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec{r}\,')}{|r-r'|}dx'dy'dz'$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{A} = \vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{A}}_{=0})-\nabla^2\vec{A} = \mu_0\vec{J}$$

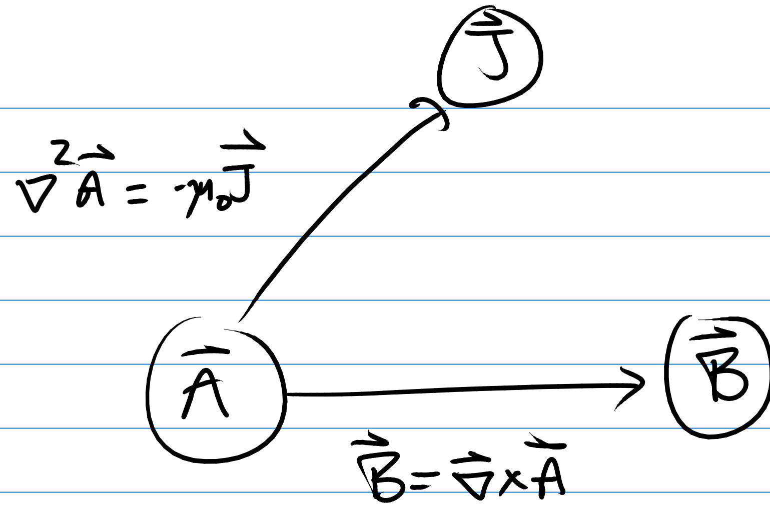
0 by choice of guage

$$\boxed{\nabla^2\vec{A} = -\mu_0\vec{J}} \quad \left\{\begin{array}{l}\nabla^2 A_x = -\mu_0 J_x\\ \nabla^2 A_y = -\mu_0 J_y\\ \nabla^2 A_z = -\mu_0 J_z\end{array}\right.$$

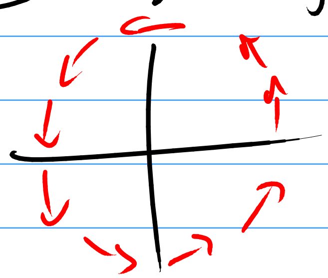
If $\vec{J}\to 0$ at ∞ (as $\rho\to 0$ at ∞)

$$A_x(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{J_x(\vec{r}\,')\,d\tau'}{|\vec{r}-\vec{r}\,'|} \quad \left\{\begin{array}{l}J_x\,d\tau'\\ K_x\,da'\\ I_x\,dl'\end{array}\right.$$

$\vec{A}$ is in direction of $\vec{J}$



DhSurvey



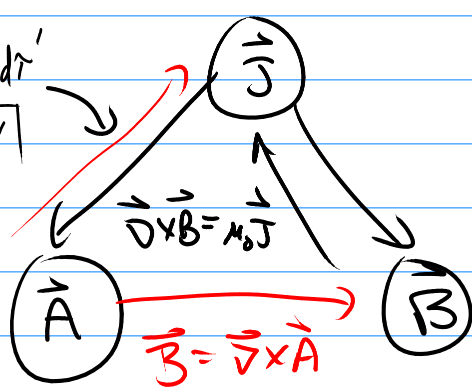
$$\vec{A} = \underbrace{\overset{v_\varphi}{k}}_{\text{const}}\,\hat{\varphi} + \overset{v_r}{0}\,\hat{r} + \overset{v_z}{0}\,\hat{z}$$

zero

$$\vec{\nabla}\times\vec{v} = \left[\frac{1}{r}\frac{\partial \overset{0}{v_z}}{\partial\varphi} - \frac{\partial \overset{k}{v_\varphi}}{\partial z}\right]\hat{r} + \left[\frac{\partial \overset{0}{v_r}}{\partial z} - \frac{\partial \overset{0}{v_z}}{\partial r}\right]\hat{\varphi} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r\overset{k}{v_\varphi}) - \frac{\partial \overset{0}{v_r}}{\partial\varphi}\right]\hat{z}$$

Strategy: go ccw around triangle diagram.
1^{st} find $\vec{B} = \vec{\nabla}\times\vec{A}$ 2^{nd} find $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ to get $\vec{J}$

$$A_x(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{J_x(\vec{r}\,')\,d\tau'}{|\vec{r}-\vec{r}\,'|}$$



$$\boxed{\nabla^2\vec{A} = -\mu_0\vec{J}}$$

$v_\varphi = k$

$v_r = 0$

$v_z = 0$

$$\underbrace{\vec{\nabla}\times\vec{A}}_{B} = \frac{1}{r}\frac{\partial}{\partial r}(rk)\hat{z} = \boxed{\frac{k}{r}\hat{z} = \vec{B}}$$

Now calculate $\vec{\nabla}\times\vec{B}$ where $v_\varphi = 0$; $v_r = 0$; $v_z = \frac{k}{r}$

$$\vec{\nabla}\times\vec{v} = \left[\frac{1}{r}\frac{\partial \overset{k/r}{\overset{\shortparallel}{v_z}}}{\partial\varphi} - \frac{\partial \overset{0}{\overset{\shortparallel}{v_\varphi}}}{\partial z}\right]\hat{r} + \left[\frac{\partial \overset{0}{\overset{\shortparallel}{v_r}}}{\partial z} - \frac{\partial \overset{k/r}{\overset{\shortparallel}{v_z}}}{\partial r}\right]\hat{\varphi} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r\overset{0}{\overset{\shortparallel}{v_\varphi}}) - \frac{\partial \overset{0}{\overset{\shortparallel}{v_r}}}{\partial\varphi}\right]\hat{z}$$

$$\vec{\nabla}\times\vec{B} = -\frac{\partial}{\partial r}\left(\frac{k}{r}\right)\hat{\varphi} = \frac{k}{r^2}\hat{\varphi}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \quad \text{so} \quad \vec{J} = \frac{\vec{\nabla}\times\vec{B}}{\mu_0} = \frac{1}{\mu_0}\frac{k}{r^2}\hat{\varphi}$$