E. Kreyszig, Advanced Engineering Mathematics, 9^{th} ed.	Section 7.1, pgs. 272-278
Lecture: Matrices, Vectors: Algebra of '+'	Module: 02
Suggested Problem Set: Suggested Problems : {5,7}	June 15, 2009

E. Kreyszig, Advanced Engineering Mathematics, 9^{th} ed.	Section 7.2, pgs. 278-287
Lecture: Matrix Multiplication: Algebra of '·'	Module: 02
Suggested Problem Set: Suggested Problems : {3, 5, 8, 13, 19, 20, 22}	June 15, 2009

Quote of Lecture 2

Remember where the thought is. I brought all this so you could survive when law is lawless.

Gorillaz: Clint Eastwood (2001)

We begin the course with the algebra of matrices. To do this we must do the following:

- Define a mathematical object called a **matrix**.
- Define how groups of **matrices** behave with respect to the binary operations '+' and '·'.

once this is completed we can begin to study how this algebra relates to linear systems and how we should think about solutions to linear problems. You have studied linear equations before in the sense that you have asked the question, for what values of x does the equation,

$$ax = b, \quad a, b \in \mathbb{R}, \tag{1}$$

have a solution? It should be straightforward to see that for a = 0 the problem has the unique solution,

$$x = \frac{b}{a}, \quad a \neq 0. \tag{2}$$

What about the case where $a = 0$? Well, that is trickier. If $b = 0$ then the value of x doesnt matter. We always have equality, there are infinitely many choice for x! However, in the case where $b \neq 0$ we have the inconsistent equation, $0 \cdot x = b \neq 0$. There are no values of x, which will satisfy the equation.

However, we are getting ahead of ourselves. The question we need to ask now is given a lot of data how can we systematically organize it? Also, once this is done how can we manipulate groups of them? Without answers to these questions we have no hope of setting up equations (1)-(2) for large data. In what follows will record the definitions of **matrices** and their **algebraic structure**.[1]

Goals

- Understand the vocabulary describing the objects and operations of linear algebra.
- Know the algebraic rules of matrix addition, scaling, multiplication, conjugation and transposition and how these rules correspond to linear transformations/systems.

Objectives

- Associate vocabulary with objects and their notations.
- Define and practice the allowed algebraic operations for matrices highlighting how this relates to to linear systems/transformations
- Record the rules of this algebra highlighting key differences with other common algebraic structures.

[1] From the perspective of abstract algebra, a field that studies collections of objects and their properties with respect to binary operators, we would call this sort of structure a non-commutative ring.