MATH348-Advanced Engineering Mathematics

MATRIX ALGEBRA

Text: 7.7-7.8

Lecture Notes: 4

Lecture Slides: N/A

Quote of Short Homework Three	
All these squawking birds won't quir	t. Building nothing, laying bricks
	The Shins : Caring is Creepy (2001)

1. Goals

The goal of this assignment is to practice determinant and matrix inversion calculations. Specifically, this will take place within the context of a so-called *LU-Decomposition*, which can be used to demonstrate fundamental properties of inverse matrices and determinants. After this assignment the student should:

• Understand how inversions and determinants of products.

2. Objectives

To achieve the previous goals the student will meet the following objectives:

- Read section 7.7-7.8 of the text book paying particular attention to pages 309-311, 317-318 and 320-322. Specifically, if the students fail to follow the in-class examples then they should work through the steps example 2 from section 7.7 and example 1 of section 7.8.
- Compute matrix inversions and determinants for specific matrices.

3. Problems

Given the following matrices,

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -5 & 12 \\ 0 & 2 & -10 \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}, \quad \mathbf{U} = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 6 \\ 0 & 0 & 2 \end{bmatrix}.$$

(1) Verify that $\mathbf{A} = \mathbf{L}\mathbf{U}$.¹

- (2) Verify that $det(\mathbf{LU}) = det(\mathbf{L})det(\mathbf{U})$.
- (3) One can show that,

$$\mathbf{A}^{-1} = \begin{bmatrix} -13 & 7 & 9/2 \\ -10 & 5 & 3 \\ -2 & 1 & 1/2 \end{bmatrix}.$$

Verify that $\mathbf{A}^{-1} = (\mathbf{L}\mathbf{U})^{-1} = \mathbf{U}^{-1}\mathbf{L}^{-1}$.

¹This is called the LU decomposition of \mathbf{A} and is useful for numerical computations with matrices. The \mathbf{L} matrix contains a 'record' of the row-steps used to arrive at the echelon-form \mathbf{U} . For more information one can see section 20.2 pages 841-842.