PHGN361 Exam 2: NAME

1. Write an integral expression for the the voltage at some arbitrary point from a continuous distribution of dipoles. **For credit explain** the meaning of the terms in the integrand and the limits of the integral.

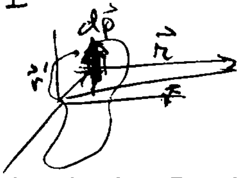
$\vec{P}$ is the $\frac{\text{dipole mom}}{\text{vol}}$

$$V = \int dV_{dipole} = \frac{1}{4\pi\epsilon_0}\int \frac{\vec{P}\cdot\hat{\mathscr{r}}\, d\tau}{\mathscr{r}^2}$$

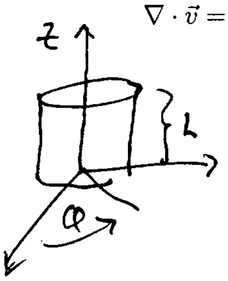
$d\vec{p} = \vec{P}\, d\tau$

$\vec{\mathscr{r}}$ from $d\vec{p}$ to field point

region that contains dipoles



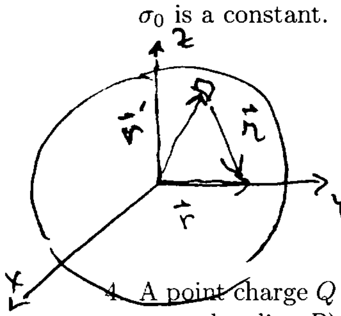
2. A cylinder of radius R and height L has dipoles glued in place so that the dipole moment per unit volume is $\vec{P} = s\hat{z}$ where s is the radial direction in cylindrical coordinates. (a) Find σ_b and ρ_b. Note $\nabla\cdot\vec{v} = \frac{1}{s}\frac{\partial}{\partial s}(sv_s) + \frac{1}{s}\frac{\partial}{\partial\phi}(v_\phi) + \frac{\partial}{\partial z}v_z$



$$\sigma_b = \vec{P}\cdot\hat{n} = \begin{cases} s & z = L \\ -s & z = 0 \end{cases}$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} = -\vec{\nabla}\cdot s\hat{z} = -\frac{\partial}{\partial z}s = 0$$

3. A sphere of radius R, made of a continuous distribution of dipoles has $\sigma_b = \sigma_0\cos\theta$ and $\rho_b = 0$ where σ_0 is a constant. Write an integral expression for V at $\vec{r} = y_0\hat{y}$.

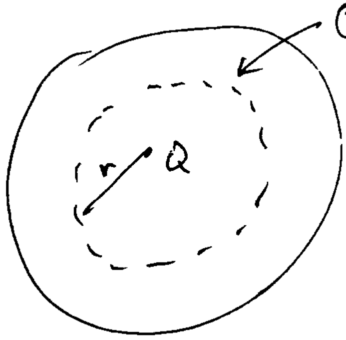


$dV = \frac{1}{4\pi\epsilon_0}\frac{dq_b}{\mathscr{r}}$ $\quad dq_b = \sigma_0\cos\theta'\, da = \sigma_0\cos\theta'\, r'^2\sin\theta'\, d\theta' d\phi'$

$\vec{\mathscr{r}} = \vec{r} - \vec{r}'$ $\quad \vec{r} = y_0\hat{y}$ $\quad \vec{r}' = r'\sin\theta'\cos\phi'\hat{x} + r'\sin\theta'\sin\phi'\hat{y} + r'\cos\theta'\hat{z}$

$r' = R$

4. A point charge Q is imbedded at the center of a sphere of linear dielectric material (with susceptibility χ_e and radius R). Derive an expression for the electric field both inside and outside the sphere.



Gaussian spherical surface

free charge

$$\oint \vec{D}\cdot d\vec{a} = Q$$

$$D\, 4\pi r^2 = Q$$

$$D = \frac{Q}{4\pi r^2} = \epsilon E$$

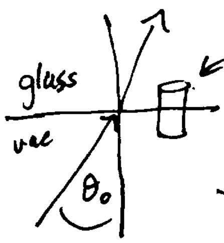
outside $D = \epsilon_0 E \Rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$

inside $D = \epsilon E \Rightarrow E = \frac{Q}{4\pi\epsilon r^2}$

$\epsilon = \epsilon_0(1+\chi_e)$

5. A plate of glass (linear dielectric of ϵ) is placed in a vacuum in the xy plane. The electric field in vacuum near the glass surface is determine (by the motion of an electron beam) to be E_0 in the xz plane at an angle of θ_0 down from the z axis. What is the magnitude and direction of the electric field in the glass if $\sigma_{free} = \sigma_0$ on the surface?

$$\tan\theta^{glass} = \frac{E_\parallel^{glass}}{E_\perp^{glass}} = \frac{E_\parallel^{vac}}{\frac{1}{\epsilon}(\sigma_0 + \epsilon_0 E_\perp^{glass})}$$



Gaussian surface

$D = \epsilon E$

$\oint \vec{D}\cdot d\vec{a} = Q_f$

$\boxed{\epsilon E_\perp^{glass} - \epsilon_0 E_\perp^{vac} = \sigma_0}$

$D_\perp^{glass}A - D_\perp^{vac}A = \sigma_0 A$

$\int \underbrace{\vec{\nabla}\times\vec{E}}_{=0}\cdot d\vec{a} = \oint \vec{E}\cdot d\vec{l} \Rightarrow \boxed{E_\parallel^{glass} = E_\parallel^{vac}}$

$$E^{glass} = \sqrt{(E_\parallel^{glass})^2 + (E_\perp^{glass})^2}$$