5-3. In order to remove a particle from the surface of the Earth and transport it infinitely far away, the initial kinetic energy must equal the work required to move the particle from $r = R_e$ to $r = \infty$ against the attractive gravitational force:

$$\int_{R_e}^{\infty} G \frac{M_e m}{r^2} dr = \frac{1}{2} m v_0^2 \tag{1}$$

where M_e and R_e are the mass and the radius of the Earth, respectively, and v_0 is the initial velocity of the particle at $r = R_e$.

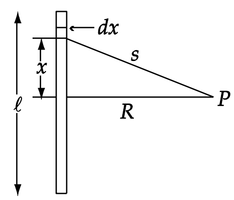
Solving (1), we have the expression for v_0:

$$v_0 = \sqrt{\frac{2G\,M_e}{R_e}} \tag{2}$$

Substituting $G = 6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2$, $M_e = 5.98 \times 10^{24}\ \text{kg}$, $R_e = 6.38 \times 10^6\ \text{m}$, we have

$$\boxed{v_0 \cong 11.2\ \text{km/sec}} \tag{3}$$

5-7.



The contribution to the potential at P from a small line element is

$$d\Phi = -G \frac{\rho_\ell}{s} dx \tag{1}$$

where $\rho_\ell = \dfrac{M}{\ell}$ is the linear mass density. Integrating over the whole rod, we find the potential

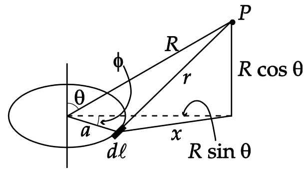
$$\Phi = -G \frac{M}{\ell} \int_{-\ell/2}^{\ell/2} \frac{1}{\sqrt{x^2 + R^2}} dx \tag{2}$$

Using Eq. (E.6), Appendix E, we have

$$\Phi = -G \frac{M}{\ell} \ln\left[x + \sqrt{x^2 + R^2} \right]\Bigg|_{-\ell/2}^{\ell/2} = -\frac{GM}{\ell} \ln\left[\frac{\dfrac{\ell}{2} + \sqrt{\dfrac{\ell^2}{4} + R^2}}{-\dfrac{\ell}{2} + \sqrt{\dfrac{\ell^2}{4} + R^2}} \right]$$

$$\boxed{\Phi = -\frac{GM}{\ell} \ln\left[\frac{\sqrt{\ell^2 + 4R^2} + \ell}{\sqrt{\ell^2 + 4R^2} - \ell} \right]} \tag{3}$$

5-10.



Using the relations

$$x = \sqrt{(R\sin\theta)^2 + a^2 - 2aR\sin\theta\cos\phi} \tag{1}$$

$$r = \sqrt{x^2 + R^2\cos^2\theta} = \sqrt{R^2 + a^2 - 2aR\sin\theta\cos\phi} \tag{2}$$

$$\rho_\ell = \frac{M}{2\pi a} \text{ (the linear mass density),} \tag{3}$$

the potential is expressed by

$$\Phi = -G\int \frac{\rho_\ell d\ell}{r} = \frac{-GM}{2\pi R}\int_0^{2\pi} \frac{d\phi}{\sqrt{1-\left[2\dfrac{a}{R}\sin\theta\cos\phi - \dfrac{a^2}{R^2}\right]}} \tag{4}$$

If we expand the integrand and neglect terms of order $(a/R)^3$ and higher, we have

$$\left[1-\left[2\frac{a}{R}\sin\theta\cos\phi - \frac{a^2}{R^2}\right]\right]^{-1/2} \cong 1 + \frac{a}{R}\sin\theta\cos\phi - \frac{1}{2}\frac{a^2}{R^2} + \frac{3}{2}\frac{a^2}{R^2}\sin^2\theta\cos^2\phi \tag{5}$$

Then, (4) becomes

$$\Phi \cong -\frac{GM}{2\pi R}\left[2\pi - \frac{1}{2}\frac{a^2}{R^2}2\pi + \frac{3}{2}\frac{a^2}{R^2}\pi\sin^2\theta\right]$$

Thus,

$$\boxed{\Phi(R) \cong -\frac{GM}{R}\left[1 - \frac{1}{2}\frac{a^2}{R^2}\left[1 - \frac{3}{2}\sin^2\theta\right]\right]} \tag{6}$$

5-14. Think of assembling the sphere a shell at a time ($r = 0$ to $r = R$).

For a shell of radius r, the incremental energy is $dU = dm\ \phi$ where ϕ is the potential due to the mass already assembled, and dm is the mass of the shell.

So

$$dm = \rho 4\pi r^2 dr = \left[\frac{3M}{4\pi R^3}\right]4\pi r^2 dr = \frac{3Mr^2 dr}{R^3}$$

$$\phi = -\frac{Gm}{r} \text{ where } m = M\frac{r^3}{R^3}$$

So

$$U = \int du$$

$$= \int_{r=0}^{R} \left[\frac{3Mr^2 dr}{R^3} \right] \left[-\frac{GMr^2}{R^3} \right]$$

$$= -\frac{3GM^2}{R^6} \int_0^R r^4 dr$$

$$\boxed{U = -\frac{3}{5} \frac{GM^2}{R}}$$