

## Exams on Feb. 7 and Feb 21

DON'T READ SECTIONS 3-2 and the first part of 3-3. Start chapter 3 at eqn M-2

Magnetic force: This lecture covers Shadowitz section 3-3 plus material not in text

Assume magneto-statics:

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t} = 0 \quad \int \vec{J}_m \cdot d\vec{a} \left( \frac{c}{s} \text{ in} \right)$$

$\rho$   $\vec{v}$

This surface has  $\frac{\partial \rho}{\partial t} \neq 0$

Questions:

no moving point charges are allowed in magnetostatics

current in wire is same everywhere even though the wire may change shape

Force law:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Lorentz force

What evidence is there that a moving q is the same as a stationary one?

What have we done?

(1) given charge or charge density we can find E and therefore the force

(2) Coulomb's law is expressed as  $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$  or  $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{\text{enc}}}{\epsilon_0}$

What are we going to do?

(1) Calculate the force given currents and B

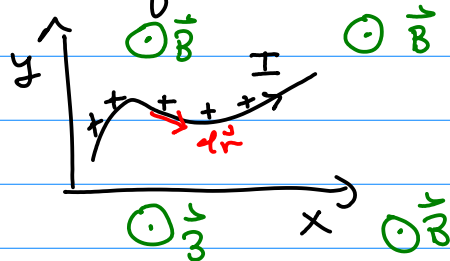
(2) Calculate B given currents

Given  $\mathbf{B}$  find force on currents using  $\vec{F} = q \vec{v} \times \vec{B}$

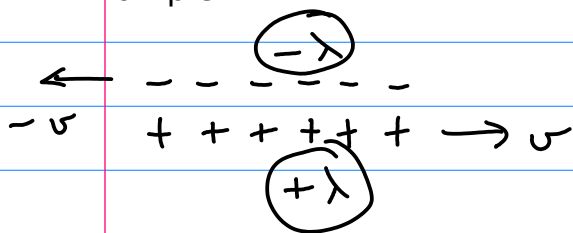
1.) charge moving along a line  $dq = \lambda |d\vec{r}|$

$$d\vec{F} = \lambda |d\vec{r}| \vec{v} \times \vec{B}$$

$$\lambda v = I \left( \frac{c}{3} \right)$$



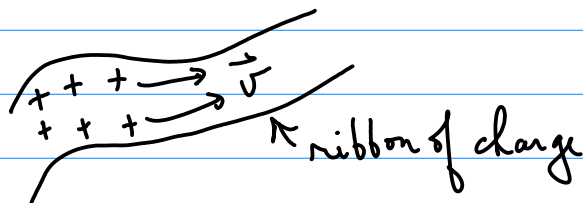
Example:



What is  $I_{total} = (-\lambda)(-v) + \lambda v = 2\lambda v$

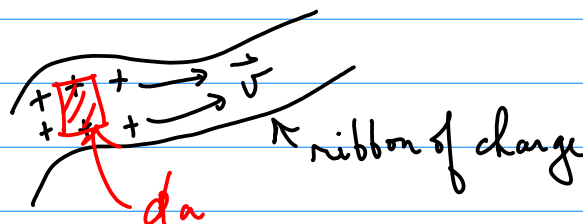
For charge moving in 1-D  $\vec{F} = \int I d\vec{r} \times \vec{B}$

2.) Charge moving along a surface.

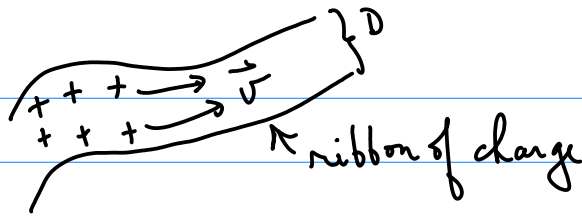


$$dq = \sigma da$$

$$d\vec{F} = dq \vec{v} \times \vec{B} = \sigma da \vec{v} \times \vec{B}$$



$$\sigma v = K \left( \frac{c}{m \cdot s} \right)$$

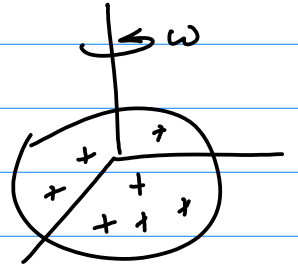


$$KD \quad \frac{C}{m \cdot s} \rightarrow I \text{ Amps} \left( \frac{C}{s} \right)$$

$$\sigma \vec{v} = \vec{K}$$

$$d\vec{F} = q\vec{v} \times \vec{B} = \vec{K} \times \vec{B} da$$

Hint: charged rod



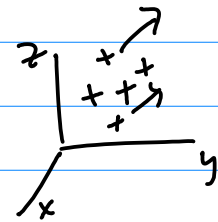
For a charge moving in 2-D

$$\vec{F} = \int \vec{K} \times \vec{B} da$$

3.) Charge moving through a volume

$$d\vec{F} = dq \vec{v} \times \vec{B}$$

$$dq = \rho d\text{volume}$$



$$d\vec{F} = \rho d\tau \vec{v} \times \vec{B}$$

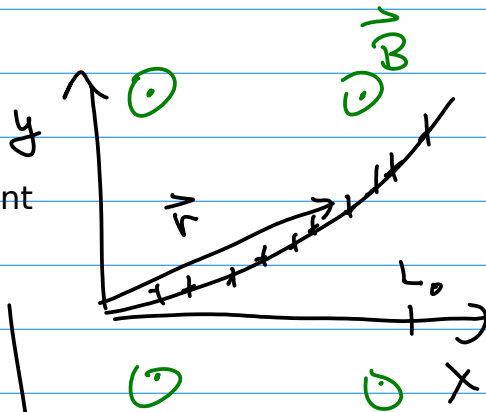
$$\vec{J} = \rho \vec{v} \text{ current density}$$

Remember  $\vec{J}$  in  $\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$

$$d\vec{F} = \vec{J} \times \vec{B} d\tau$$

Example:

Find the force on the wire segment from  $x=0$  to  $x=L$



$$d\vec{F} = I d\vec{r} \times \vec{B}$$

$$\vec{r} = x\hat{x} + x^2\hat{y}$$

$$d\vec{r} = dx\hat{x} + 2x dx\hat{y}$$

$$d\vec{F} = I \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ dx & 2x dx & 0 \\ B_x & B_y & B_z \end{vmatrix}$$

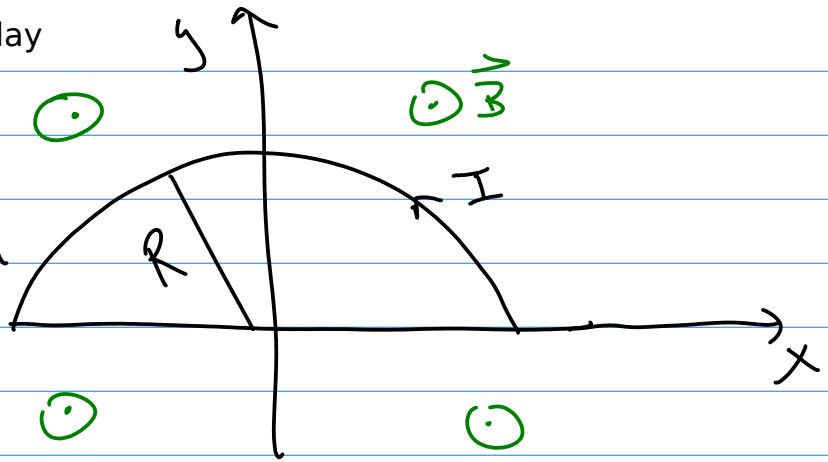
expand and integrate

For a charge moving in 3-D

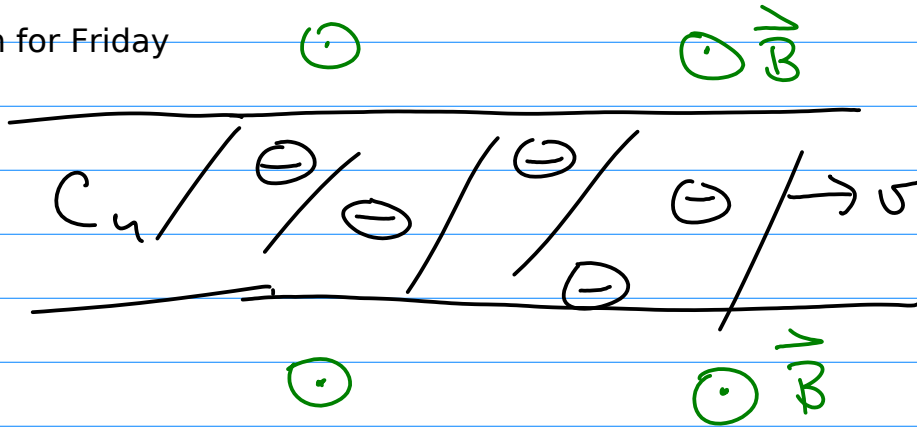
$$\vec{F} = \int \vec{J} \times \vec{B} d\tau$$

InkSurvey question for Friday

Find  $d\vec{l} = d\vec{r}$  needed to determine  $d\vec{F}$



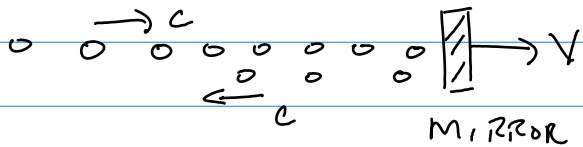
InkSurvey question for Friday



How can there be a force on a wire if the electrons are free to move?

**Example: conservation of photons in a laser beam reflecting from a moving mirror**

Photons reflecting as a stream of particles 1-D



$$\vec{J}_{in} = \rho_{in} \vec{v}_{in} \rightarrow \lambda_{in} c$$

$$\vec{J}_{reflected} = \rho_{ref} \vec{v}_{ref} \rightarrow \lambda_{ref} c$$

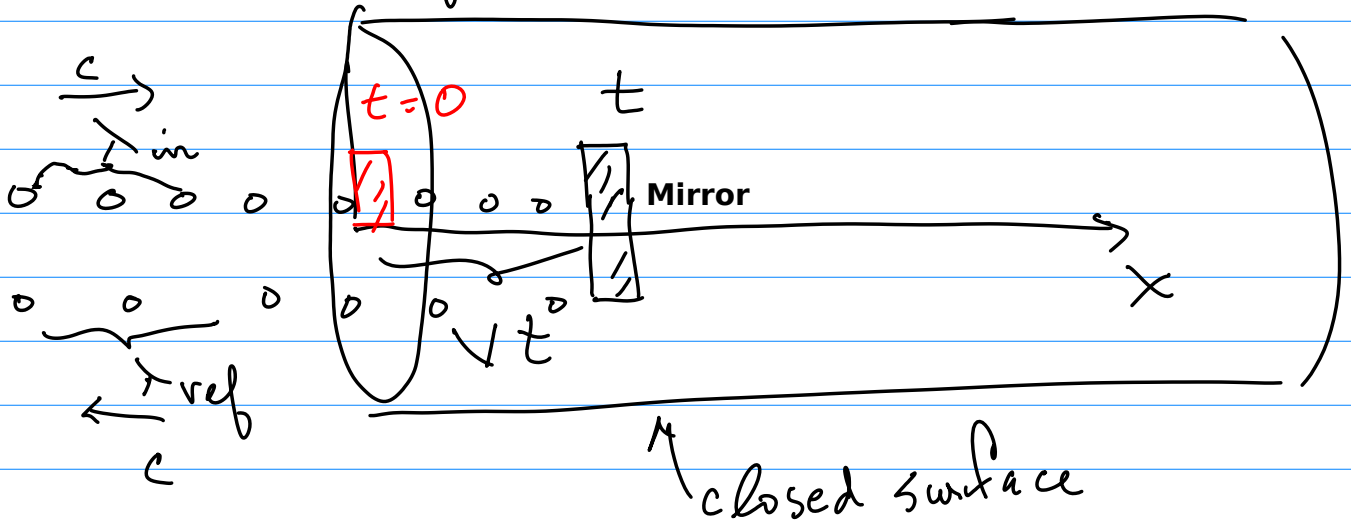
Model

$$\vec{J}_{in} = \lambda_{in} c \hat{x}$$

$$\vec{J}_{ref} = -\lambda_{ref} c \hat{x}$$

Reality

$$10^{14} \frac{\text{photons}}{\text{sec}} \pm \sqrt{10^{14}}$$



Conservation eqn?

$$J_{in} - J_{ref} = \frac{dN_{inside}}{dt}$$

↑ flux of particles into surface  
 ↑ flux of particles out surface  
 ← time rate of change of # inside

$$\lambda_{in} c - \lambda_{ref} c = \frac{d}{dt} (\lambda_{in} + \lambda_{ref}) V t = (\lambda_{in} + \lambda_{ref}) V$$

$$\lambda_{ref} = \frac{c-v}{c+v} \lambda_{in} = \frac{c(1-v/c)}{c(1+v/c)} \lambda_{in} \approx \lambda_{in} (1 - \frac{2v}{c})$$

$$J_{ref} = J_{in} (1 - \frac{2v}{c})$$

Series approx in  $\delta = \frac{v}{c}$   
 Use Series in Mathematica

Photodetector (photodiode or photomultiplier) measures photon flux

## Questions:

### Incongruous:

*How can this be correct since light is a wave?*

### Congruous:

*How do I apply the continuity equation if the reflection is not normal to the surface of the mirror?*

### Modifying:

*How do I calculate conservation of photon number if the mirror accelerates?*

*How does the calculation change if particles with non-zero rest mass are used?*

### Generalizing/Analogy:

*How does this model match the classical electromagnetic wave model?*

### Causal/Creative:

*How do I change from conservation of number to conservation of energy?*

*How do I change from conservation of number to conservation of momentum?*

### Informational:

*How many photons per second are emitted by a He-Ne laser?*