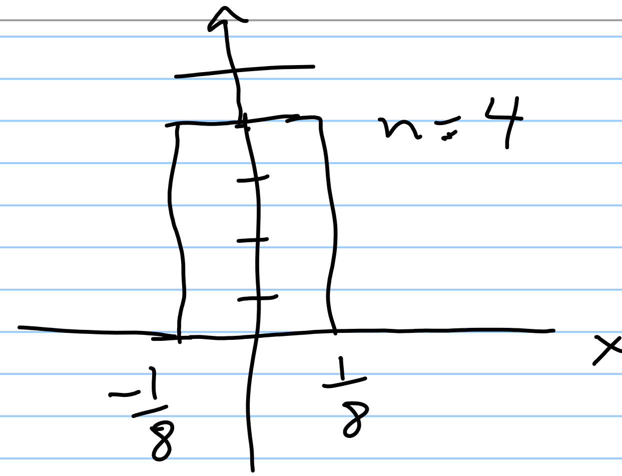
Delta function

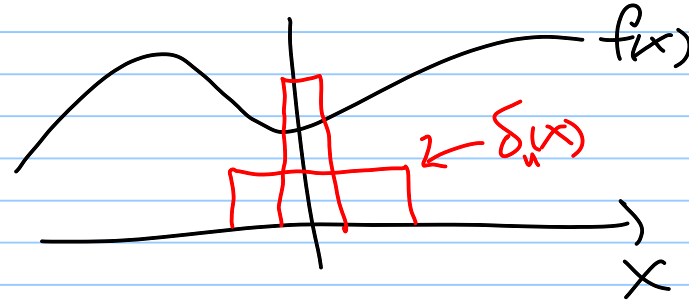
$$\delta_n(x) = \begin{cases} 0 & x < \frac{-1}{2n} \\ n & \frac{-1}{2n} < x \quad \frac{1}{2n} \\ 0 & x > \frac{1}{2n} \end{cases}$$



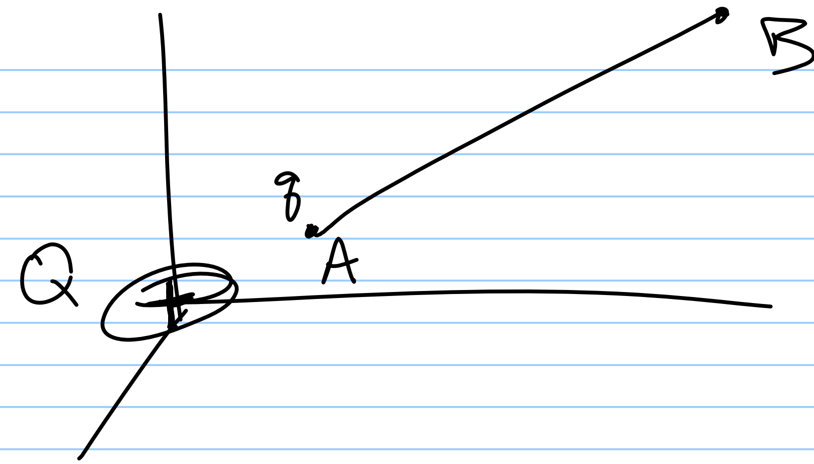
$$\text{area} = n \, \frac{1}{n} = 1 \quad \text{but } \delta_n(x) \to \infty \text{ as } n \to \infty$$

$$\lim_{n \to \infty} \int f(x)\, \delta_n(x)\, dx$$

$$= f(0) \underbrace{\int \delta_n(x)\, dx}_{1}$$



Ex:



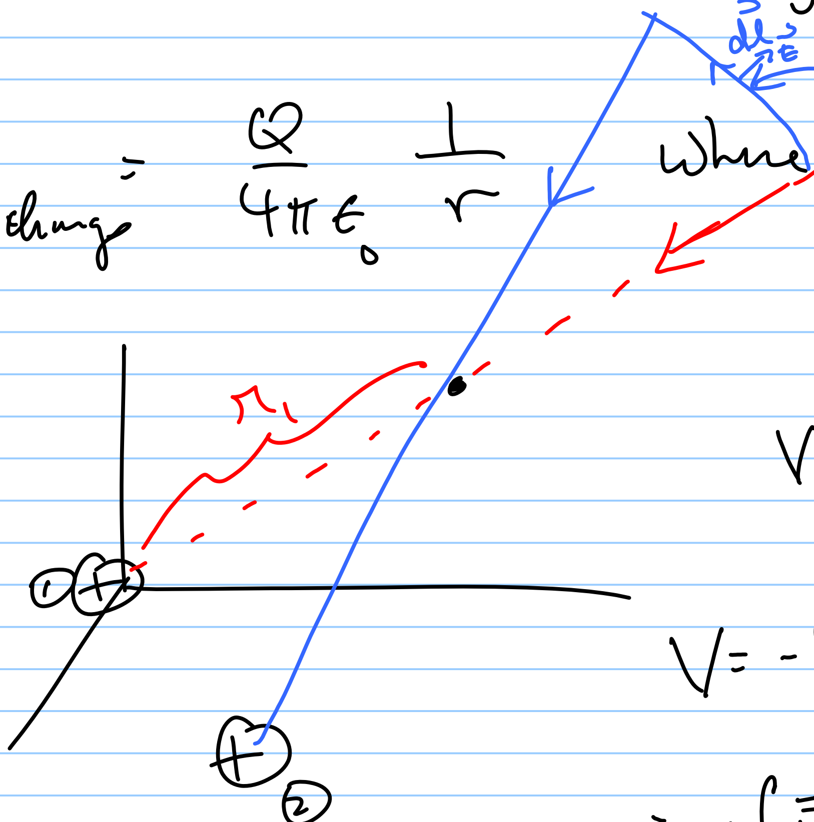
$$W_{cons} = \int \vec{F}_{coul} \cdot d\vec{\ell} = \int_A^B q \frac{kQ}{r^2} \hat{r} \cdot dr\,\hat{r}$$

$$= qkQ \int_A^B \frac{dr}{r^2} = qkQ\left(-\frac{1}{r}\right)\Big|_A^B = qkQ\left(\frac{-1}{r_B} + \frac{1}{r_A}\right)$$

$$W_{cons} = \underline{q}\, \underbrace{\frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)}_{\Delta V} = -\Delta PE = \underline{q} \underbrace{\int \vec{E} \cdot d\vec{\ell}}_{-\Delta V}$$

$\vec F = q\vec E$ ← field to other charges

$W_{cons} = q\Delta V$ $\quad\quad \Delta V \equiv -\int \vec E \cdot d\vec l$



$\vec E \cdot d\vec l = 0$

$V_{pt\ charge} = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}$ where $\underline{V(r=\infty) \equiv 0}$

$$V = -\int \vec E \cdot d\vec l$$

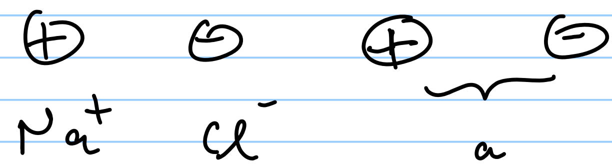
$$V = -\int\left(\vec E_{1} + \vec E_{2}\right)\cdot d\vec l$$

$$= \underbrace{-\int \vec E_{1}\cdot d\vec l}_{\frac{1}{4\pi\epsilon_0}\frac{Q_1}{r_1}} \underbrace{-\int \vec E_{2}\cdot d\vec l}_{+\ \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r_2}}$$

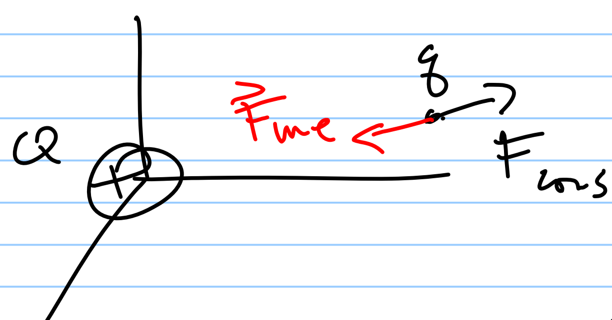
$$V = \sum_{i=1}^{N} \frac{k Q_i}{r_i} \longrightarrow \int \frac{k\,dq}{r_i}$$

∞ at ∞

W_{cons} in bring in q is $q\Delta V = q(V_f - \overset{\shortparallel}{V_i})$



W_{me} in assembling this "crystal"



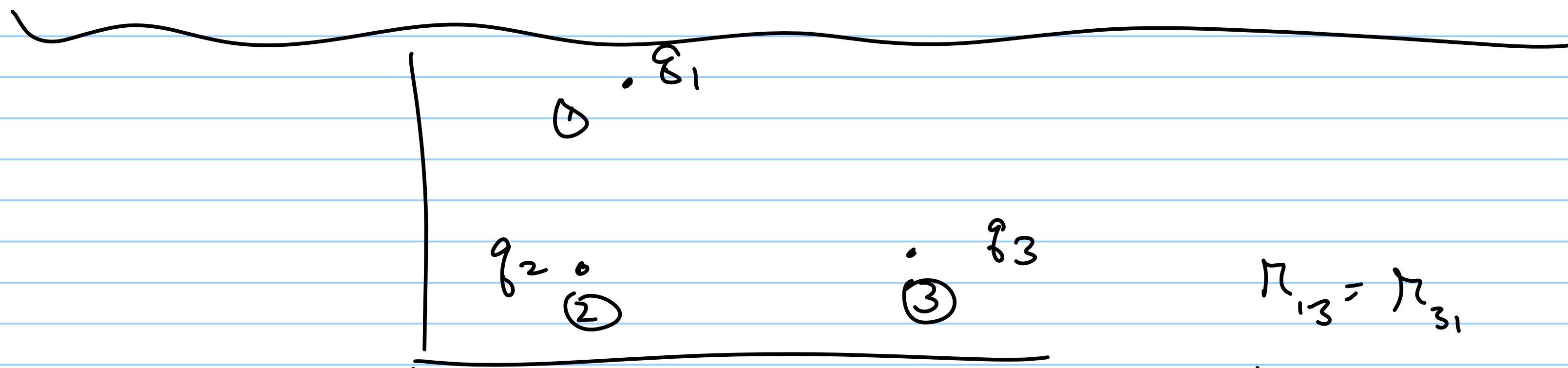
$$W_{cons} = \int \vec{F}_{cons} \cdot d\vec{\ell} = -\Delta PE$$

$$= \int q \vec{E}_{cons} \cdot d\vec{\ell}$$

$$W_{noncon} = \Delta(\underbrace{KE}_{=0} + PE)$$

$$W_{me} = \Delta PE = -W_{cons}$$

I oppose electric force



$$r_{13} = r_{31}$$

$$r_{23} = r_{32}$$

$$W = k\frac{q_2 q_1}{r_{12}} + k\frac{q_3 q_2}{r_{32}} + k\frac{q_3 q_1}{r_{31}}$$

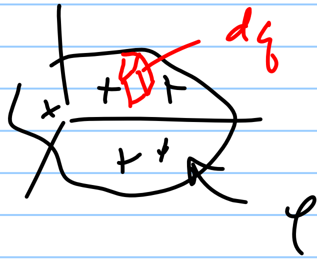
$$W = \frac{1}{2}\left\{ \frac{q_1}{4\pi\epsilon_0}\left(\frac{q_2}{r_{12}} + \frac{q_3}{r_{13}}\right) + \frac{q_2}{4\pi\epsilon_0}\left(\frac{q_1}{r_{12}} + \frac{q_3}{r_{23}}\right) + \frac{q_3}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)\right\}$$

$$= \frac{1}{2}\left(q_1 V(\text{Point}_1) + q_2 V(\text{Point}_2) + q\, V(\text{Point}_3)\right)$$

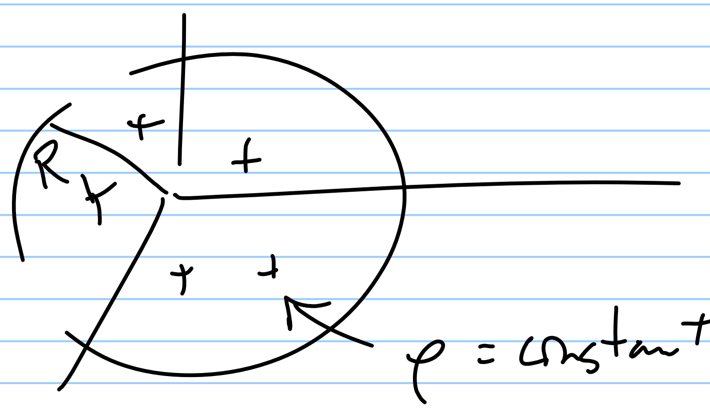
with all other charges present

$$W = \frac{1}{2}\sum{}' q_i V_i \longrightarrow \frac{1}{2}\int V\,dq$$

voltag when all charges are present

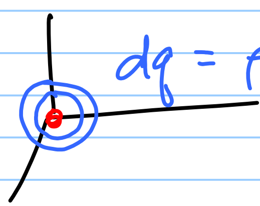


Ex of the 1st method



W_{me} to assemble this sphere of charge

Principles



$dq = \rho\, d\tau = \rho\, 4\pi r^2 dr$

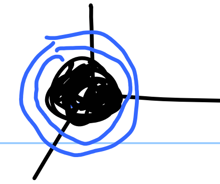
$$W_{me} = q\,\Delta V = q\,V$$

final location of charge q

$$dW = dq\, V$$

$$W_{me} = \int V \, dq$$

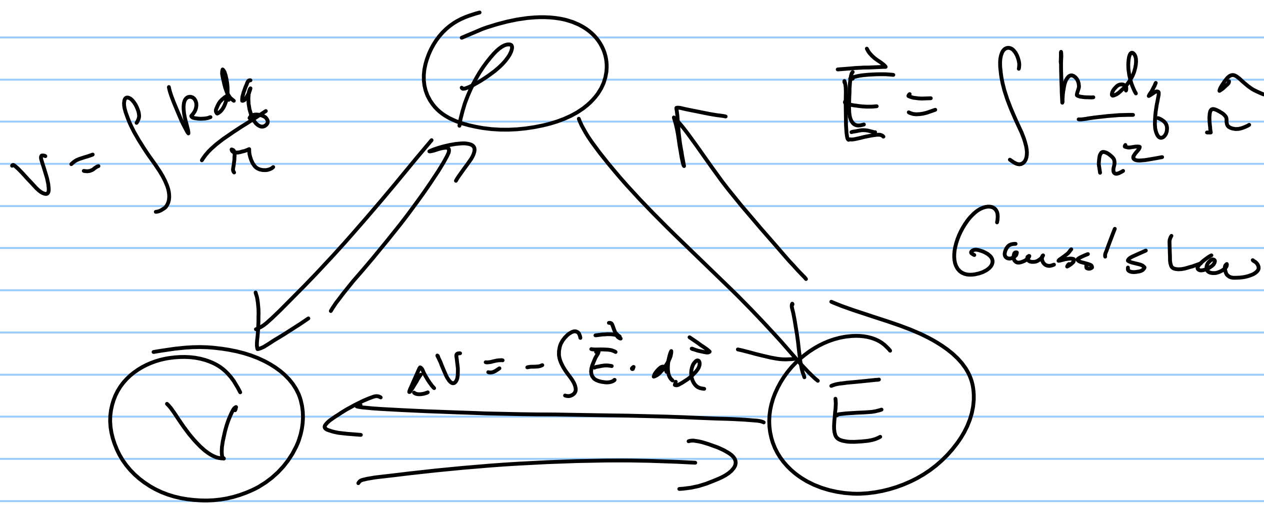
voltage when only the charges assembled at that point are in



$$V = -\int \vec{E} \cdot d\vec{l}$$

$$W_{me} = \int_0^R \frac{1}{4\pi\epsilon_0} \frac{\rho \frac{4}{3}\pi r^3}{r} \rho \, 4\pi r^2 dr = \frac{1}{4\pi\epsilon_0} \frac{3}{5} \frac{q^2}{R}$$

$$q = \rho \frac{4}{3}\pi R^3$$



$$\vec{F} = q\vec{E} = m\vec{a}$$

$$W_{nc} = \Delta(KE + PE)$$

$$\Delta PE = q\Delta V$$