Problem Set 1

Due: Tuesday, June 26th, at the *beginning* of class.

1. Calculate the first two terms for the electric field (E_2 and E_4) and the first three terms in the magnetic field (B_1, B_3, B_5) in the perturbative expansion for the parallel plate capacitor done in class (harmonic time dependance).
2. Chapter 7: 34, 35, 58, 60 (a)
3. Chapter 8: 1, 2, 4, 5, 6, 7, 11, 12

Extra Credit

The zeroth order Bessel function is defined by the power series

$$J_0(\lambda z) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!m!} \left(\frac{\lambda z}{2}\right)^{2m}$$

and satisfies the ODE

$$z\frac{d}{dz}\left(z\frac{dJ_0}{dz}\right) + \lambda^2 z^2 J_0 = 0.$$

1. Prove that the full solution for electric field of the parallel plate capacitor from question 1 is

   $$\vec{E}(s,t) = E_0 e^{i\omega t} J_0(\omega s/c)\hat{z}.$$

   I can think of at least two ways to do this.

   Method 1: Determine the 2nd order ODE that the spatial part of electric field satisfies and show that it is the same as that of the zeroth order Bessel function.

   Method 2: Try to come up with a form for the nth term for the expansion of the spatial part of the electric field. Next you can show that the recursion relation between the nth term and the $(n-1)$th term is the same as that for the zeroth order Bessel function. Additionally, if the two series start with the same first term, and have the same recursion for the subsequent terms, then the two series are the same.
2. Using the fact that

   $$J_1(z) = -\frac{dJ_0(z)}{dz},$$

   show that the solution for the magnetic field of the capacitor can be written as

   $$\vec{B}(s,t) = \frac{iE_0}{c} e^{i\omega t} J_1(\omega s/c)\hat{z}.$$

   Notice that the factor of i means that the magnetic field will be 90° (or $\pi/2$) out of phase (temporally) with the electric field. By this I am saying that if the time dependence of the electric field is a cos, then the time dependence of the magnetic field will be a -sin. So when one is a maximum, the other is zero. This came about because Maxwell's Equations relate each field to the time derivative of the other's field.