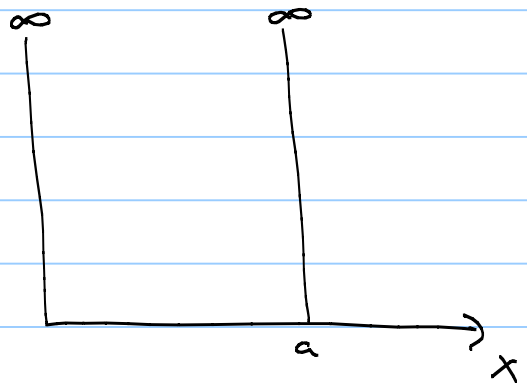
Sample questions

Note Title 1/31/2008

(1)



a particle of mass m is confined to the potential at the left.

At $t=0$ its Normalized wave function is

$$\psi(x, t=0) = \sqrt{\frac{8}{5a}} \sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{2}{5a}} \sin\left(\frac{2\pi x}{a}\right)$$

a) what is the wavefunction at a later time $t = t_0$?

b) what is the average energy of the system at $t=0$ and $t = t_0$?

c) what is the probability that the particle is found in the left half of the box at $t = t_0$?

(2) A electron is confined in the ground state in a 1-D box as in problem (1) with width $a = 10^{-10}$ m. Its energy is 38 eV.

Calculate the energy of the first excited state

(3) Consider the wavefunction

$$\psi(x) = A (x/x_0)^2 e^{-x/x_0}$$

using Schrödinger's equation find the potential $V(x)$ and Energy E for which $\psi(x)$ is an eigenfunction of the Hamiltonian, assuming that $V(x) \to 0$ as $x \to \infty$.

(4) Consider a harmonic oscillator with normalized ground and first excited state wavefunctions ψ_0, ψ_1. Let A, B be real numbers and Let $A\psi_0 + B\psi_1$ be the wavefunction at some particular time. Show that $\langle x \rangle$ cannot be zero.

(5) Let $a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + \frac{i}{m\omega}p\right)$

$a^+ = \sqrt{\frac{m\omega}{2\hbar}}\left(x - \frac{i}{m\omega}p\right)$

be the lowering and raising operators for the QHO.

Prove that $a^+a = \frac{1}{\hbar\omega}H - \frac{1}{2}$

where $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$ is the Hamiltonian.

(6) a) Prove that $[x, p] = i\hbar$

b) For the QHO what is the eigenvalue for the n-th stationary state ψ_n?

c) Prove that $\int_{-\infty}^{\infty}\left[\frac{\partial^2\psi^*}{\partial x^2}\psi - \psi^*\frac{\partial^2\psi}{\partial x^2}\right]dx = 0$

(7) a) Why are stationary states called stationary?

b) Prove that if a system

is initialized in an E-state of H, it stays in that state indefinitely.

(8) What is a deBroglie wave? explain why such waves are not physically realizable. what is σ_p for such a wave function?

(9) Let $\psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]}$.

a) Find A.

b) Calculate σ_x and σ_p and show that $\sigma_x \sigma_p$ is indeed $\geq \frac{\hbar}{2}$.