

near 4.1 - 4.3 today
4.4 - 4.5 wed.

Tensor components of χ_{ijkl} : isotropic medium

only 4 components:

$$\chi_{nn} (= \chi_{2222} = \chi_{3333})$$

χ_{1122} (+ permutations of pairs e.g. χ_{2233})

χ_{1212} (permute)

χ_{1221} ("")

recall classical model:

$$\vec{F} = -m\omega_0^2 \vec{r} + mb(\vec{r}, \vec{r})\vec{r}$$

$$\sim -K_{\text{eff}} \vec{r} \quad \text{w/ } K_{\text{eff}} = m(\omega_0^2 - b \vec{r}, \vec{r})$$

$\vec{F} \parallel$ to one input \vec{A} & \vec{d}

other 2 must be \parallel

$$\leftrightarrow \begin{matrix} K_{\text{eff}} \sim m(\omega_0^2 - b) \\ \vec{F} \sim \vec{\chi} \end{matrix}$$

$$\text{Also, } \chi_{nn} = \chi_{1122} + \chi_{1212} + \chi_{1221}$$

(isotropic; must have same response if material rotates by 45°)

Intrinsic permutation symm: exchange cartesian index and w's

$$\chi_{1122} = \chi_{1212} + \chi_{1221}$$

$$\begin{pmatrix} w_1 = w_1 + w_2 - w_3 \\ 1 \quad 2 \quad 3 \end{pmatrix} \quad \rightarrow \text{since last } w \text{ is } -w$$

We can write full matrix as

$$\chi_{ijkl} = \chi_{1122} \delta_{ij} \delta_{kl} + \chi_{1212} \delta_{ik} \delta_{jl} + \chi_{1221} \delta_{il} \delta_{jk}$$

↓
cyclic ↓

$$P_n(w) = D \epsilon_0 \sum_{jkl} \chi_{ijkl} (w = w_1 + w_2 - w_3) E_j(w) E_k(w) E_l(-w)$$

D = # distinct perm.

of w's

here D=3
 $w_1, w_2 - w_3, w_1, w_2, w_3$

$$\rightarrow 3 \epsilon_0 \sum_{jkl} \left(\chi_{nnn} (\delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl}) + \chi_{1122} \delta_{il} \delta_{jk} \right) E_j E_k E_l^*$$

$$\frac{1}{E_i E_j} \frac{1}{E_k E_l} \rightarrow \vec{E} \cdot \vec{E}^*$$

group terms:

$$\vec{P} = 6\epsilon_0 \chi_{1122} (\vec{E} \cdot \vec{E}^*) \vec{E} + 3\epsilon_0 \chi_{1221} (\vec{E} \cdot \vec{E}) \vec{E}^*$$

define $A = 6\chi_{1122}$ $B = 3\chi_{1221}$



different mechanisms lead to diff't ratios of B/A
 $B/A = 1$ for non-resonant electronic response.

polarization dependence:

linear: $\vec{E} = E_0 \hat{x} \rightarrow \vec{P} = (A + \frac{1}{2}B) |E|^2 \vec{E}$

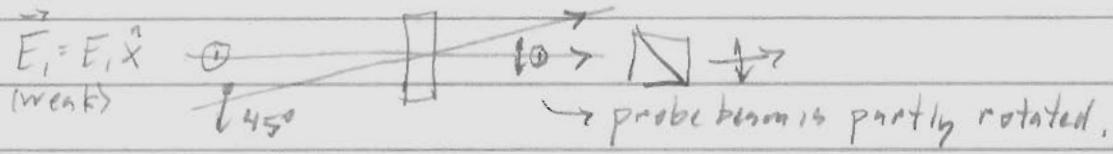
circular: $\hat{\sigma}_{\pm} = \frac{1}{\sqrt{2}} (\hat{x} \pm i\hat{y})$

$$|\hat{\sigma}_{\pm}|^2 = 1 \quad \text{but} \quad \hat{\sigma}_+ \cdot \hat{\sigma}_{\pm} = 0$$

$$\therefore \vec{P} = A |E|^2 \vec{E} \quad \text{lower } m_z \text{ for circ. pol.}$$

Polarization gating

2 crossed input beams, one at 45° polarization.



$$\vec{E}_2 = E_2 \frac{1}{\sqrt{2}} (\hat{x} + \hat{y}) \quad (\text{strong})$$

output E_y component: $\sim P_y$

$$P_y \sim A(\vec{E} \cdot \vec{E}^*) E_y + \frac{1}{2} B(\vec{E} \cdot \vec{E}) E_y^* \quad E_y \text{ is only from } E_1$$

$$\sim A|E_1|^2 E_y + A \frac{1}{2}|E_2|^2 E_y + A E_1 E_2^* E_y + A E_1^* E_2 E_y$$

~~$\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$~~

gated

$$+ \frac{1}{2} B E_1^2 E_y^* + \frac{1}{2} B \frac{E_2^2}{2} E_y^* + \frac{1}{2} B 2 E_1 E_2 E_y^*$$

$$\underbrace{\qquad\qquad\qquad}_{\text{gated.}} e^{i k_1 r} \cdot e^{i k_2 r} e^{-i k_3 r}$$

Applications:

- pulse characterization $|E_2|^2$ is the gate
- Kerr optical shutter.

Alternative: $P_g(w) = 6 \epsilon_0 \sum_{jkl} (\chi_{1zz2} \delta_{ij} \delta_{kl} + \chi_{1zz2} \delta_{ik} \delta_{jl} + \chi_{1zz1} \delta_{il} \delta_{jk})$
 w along beam 1

w' along beam 2 $j \neq z, i \neq l$

$$P_g(w) = 6 \epsilon_0 \frac{|E_2|^2}{2} E_1 (\chi_{1zz2} + \chi_{1zz1})$$

$$E_j(w) E_k(w') E_l(-)$$

NL refractive index (and other degenerate $\chi^{(3)}$ response)

4-wave mixing: $\omega_4 = \omega_1 + \omega_2 + \omega_3$ (ω 's are > 0 or < 0)

Degenerate FWM:

$$\omega = \omega_1 + \omega_2 - \omega_3$$

looks like it doesn't do anything, but:

→ phase change: NL refr. index $n(I) = n_0 + n_2 I$

$$\rightarrow \phi(I) = \omega_0 n_2 I \propto$$

• self phase modulation (SPM), spectral broadening

• self focusing

→ polarization rotation

→ cross-polarized wave generation (XPG)

Representations

field $n = n_0 + \bar{n}_2 \langle \tilde{E}^2 \rangle = n_0 + 2\bar{n}_2 |E(w)|^2$

intensity $n = n_0 + n_2 I$

NL polarization, $\chi^{(3)}$

$$P^{NL}(w) = D \epsilon_0 \chi^{(3)} |E(w)|^2 E(w)$$

$D=3$ for all same beams

see connections btw them in Boyd, 4.1

- be careful about definitions + units!

* when there are 2 beams, e.g. ~~||~~ or $w_1 \nearrow \square$

then n_2 is multiplied by 2x

$D=6$ (extra 2x b/c $w_1 + w_2$)

Time response + magnitude of n_2 slower allows longer n_2

• electronic (< fs, small n_2) • semiconductor

• molecular ~ ps, larger • relativistic

• thermal ~ ms

NL polarization rotation

When a NL material is exposed to elliptical light, the material has an induced optical activity.

We can describe any polarization state as a linear combination of R ($\hat{\sigma}_-$) or L ($\hat{\sigma}_+$) circ. polarization states:

$$\vec{E} = E_+ \hat{\sigma}_+ + E_- \hat{\sigma}_- \quad \hat{\sigma}_{\pm} = \frac{1}{\sqrt{2}}(\hat{x} \pm i\hat{y})$$

our NL polarization is note that $\hat{\sigma}_+^* = \hat{\sigma}_-$

$$\vec{P}^{NL} = A(\vec{E} \cdot \vec{E}^*) \vec{E} + \frac{1}{2}B(\vec{E} \cdot \vec{E}) \vec{E}^*$$

express both sides in terms of $\hat{\sigma}_+$, $\hat{\sigma}_-$ vectors

$$P_+ \hat{\sigma}_+ + P_- \hat{\sigma}_- = A(|E_+ \hat{\sigma}_+ + E_- \hat{\sigma}_-|^2) (E_+ \hat{\sigma}_+ + E_- \hat{\sigma}_-) + \frac{1}{2}B(E_+ \hat{\sigma}_+ + E_- \hat{\sigma}_-) \cdot (E_+ \hat{\sigma}_+^* + E_- \hat{\sigma}_-^*)$$

$$\hat{\sigma}_{\pm}^* = \hat{\sigma}_{\mp}$$

$$\hat{\sigma}_+^* \cdot \hat{\sigma}_-^* = 0 \quad (\text{orthogonal})$$

$$\hat{\sigma}_{\pm}^* \cdot \hat{\sigma}_{\pm}^* = 1 \quad (\text{normalized})$$

grouping terms:

$$\begin{aligned} P_+ &= A(|E_+|^2 + |E_-|^2) E_+ + B E_+ E_- E_-^* \\ &= A|E_+|^2 E_+ + (A+B)|E_-|^2 E_+ = (A|E_+|^2 + (A+B)|E_-|^2) E_+ \\ &\equiv \chi_{+}^{NL} E_+ \end{aligned}$$

$$\begin{aligned} P_- &= A(|E_+|^2 + |E_-|^2) E_- + B E_+ E_- E_+^* \\ &= A|E_-|^2 E_- + (A+B)|E_+|^2 E_- = [A|E_-|^2 + (A+B)|E_+|^2] E_- \\ &\equiv \chi_{-}^{NL} E_- \end{aligned}$$

so the NL interaction separates:

each R, L component exper. a different effective n:

$$n_{\pm}^2 = n_0^2 + \dots \chi_{\pm}^{NL}$$

$$\Rightarrow \text{diff're } v_{ph} \quad \Delta n = n_+ - n_- = \frac{B}{2n_0} (|E_-|^2 - |E_+|^2)$$

ellipse rotation
 in \hat{B}_\pm basis, input is $\begin{pmatrix} A_+ \\ A_- \end{pmatrix}$
 $n_\pm \approx n_0 + \frac{1}{2n_0} (A|E_\pm|^2 + (A+B)|E_\mp|^2)$
 output is $\exp(i k_0 n_\pm L) E_\pm$

each wave is shifted by $k_0 \cdot \underbrace{\frac{1}{2}(n_+ + n_-)z}_{k_m} \pm \underbrace{\frac{1}{2}\Delta n k_0 z}_{\theta}$

output is then

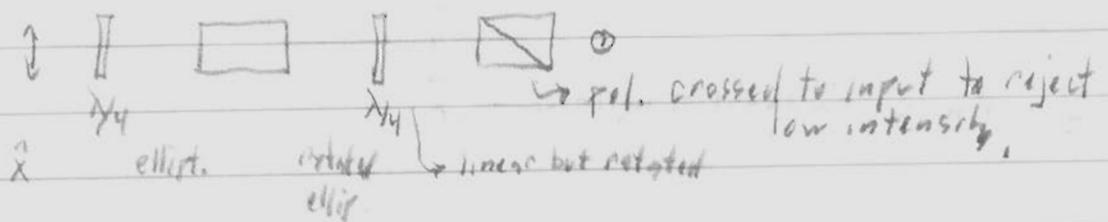
$$\vec{E}(z) = \begin{pmatrix} A_+ e^{i\theta} \\ A_- e^{-i\theta} \end{pmatrix} e^{ik_m z}$$

and the coordinate system for the ellipse rotates.

This is a kind of induced optical activity,
 but linear and circular polarization are not affected.

Applications:

contrast enhancement (self-gating)



NL ellipse rot mode locking - saturable absorber.

same config. but add $\Delta\theta$ to rotate output to peak peak intensity.