2.1.9 Let P stand for "the integer x is even," and let Q stand for "x^2 is even." Express the conditional statement $P \to Q$ in English using

1. The "if then" form of the conditional statement
2. The word "implies"
3. The "only if" form of the conditional statement
4. The phrase "is necessary for"
5. The phrase "is sufficient for"

2.3.4 Use set builder notation to specify the following sets:

1. The set of all integers greater than or equal to 5.
2. The set of all even integers.
3. The set of all positive rational numbers.
4. The set of all real numbers greater than 1 and less than 7.

2.3.5 For each of the following sets, use English to describe the set or use the roster method to specify all of the elements of the set.

1. $\{x \in \mathbb{R} \,|\, -3 \le x \le 5\}$
2. $\{x \in \mathbb{Z} \,|\, -3 \le x \le 5\}$
3. $\{x \in \mathbb{R} \,|\, x^2 = 16\}$

2.4.6 In calculus, we define a function f to be **continuous** at a real number a provided that for every $\varepsilon > 0$, there exists a $\delta > 0$ such that if $|x - a| < \delta$, then $|f(x) - f(a)| < \varepsilon$.
Complete each of the following sentences using the appropriate symbols for quantifiers:

(a) A function f is continuous at the real number a provided that ...

(b) A function f is not continuous at the real number a provided that ...

Complete the following sentence in English without using symbols for quantifiers:

(c) A function f is not continous at the real number a provided that ...

2.4.7 Let A be a subset of the real numbers. A number b is called an **upper bound** for the set A provided that for each element x in A, $x \le b$.
Also, let A be a subset of $\mathbb{R}$. A real number α is the **least upper bound** for A provided that α is an upper bound of A, and if β is an upper bound of A, then $\alpha \le \beta$.
If we define $P(x)$ to be "x is an upper bound of A", then we can write the definition for least upper bound as follows:
A real number α is the **least upper bound** for A provided that

$$P(\alpha) \wedge [(\forall \beta \in \mathbb{R})(P(\beta) \to (\alpha \le \beta)]$$

(a) Why is a universal quantifier used for the real number β?

(b) Complete the following sentence in synmbolic form: "A real number α is not the least upper bound for A provided that ...

(c) Complete the following sentence as an English sentence: "A real number α is not the least upper bound for A provided that ...

3.1.11 Let r be a positive real number. The equation for a circle of a radius r whose center is the origin is $x^2 + y^2 = 1$.

1. Use implicit differentiation to determine $\dfrac{dy}{dx}$
2. Let (a, b) be a point on the circle with $a \neq 0$ and $b \neq 0$. Determine the slope of the line tangent to the circle at the point (a, b).
3. Prove that the radius of the circle to the point (a, b) is perpendicular to the line tangent to the circle at the point (a, b).

3.1.12 Determine if each of the following statements is true or false. Provide a counterexample for statements that are false and provide a complete proof for those that are true.

1. For all real numbers x and y, $\sqrt{xy} \leq \dfrac{x+y}{2}$.
2. For all real numbers x and y, $xy \leq \left(\dfrac{x+y}{2}\right)^2$.
3. For all nonnegative real numbers x and y, $\sqrt{xy} \leq \dfrac{x+y}{2}$.

3.1.13 Use one of the true inequalities in Exercise (12) to prove the following proposition.

For each real number, a, the value of x that gives the maximum value of $x(a - x)$ is $x = \dfrac{a}{2}$.

3.2.10 Prove that for each integer a, if $a^2 - 1$ is even, then 4 divides $a^2 - 1$.

3.2.17 Prove the following proposition:

Let a and b be integers with $a \neq 0$. if a does not divide b, then the equation $ax^3 + bx + (b + a) = 0$ does not have a solution that is a natural number.