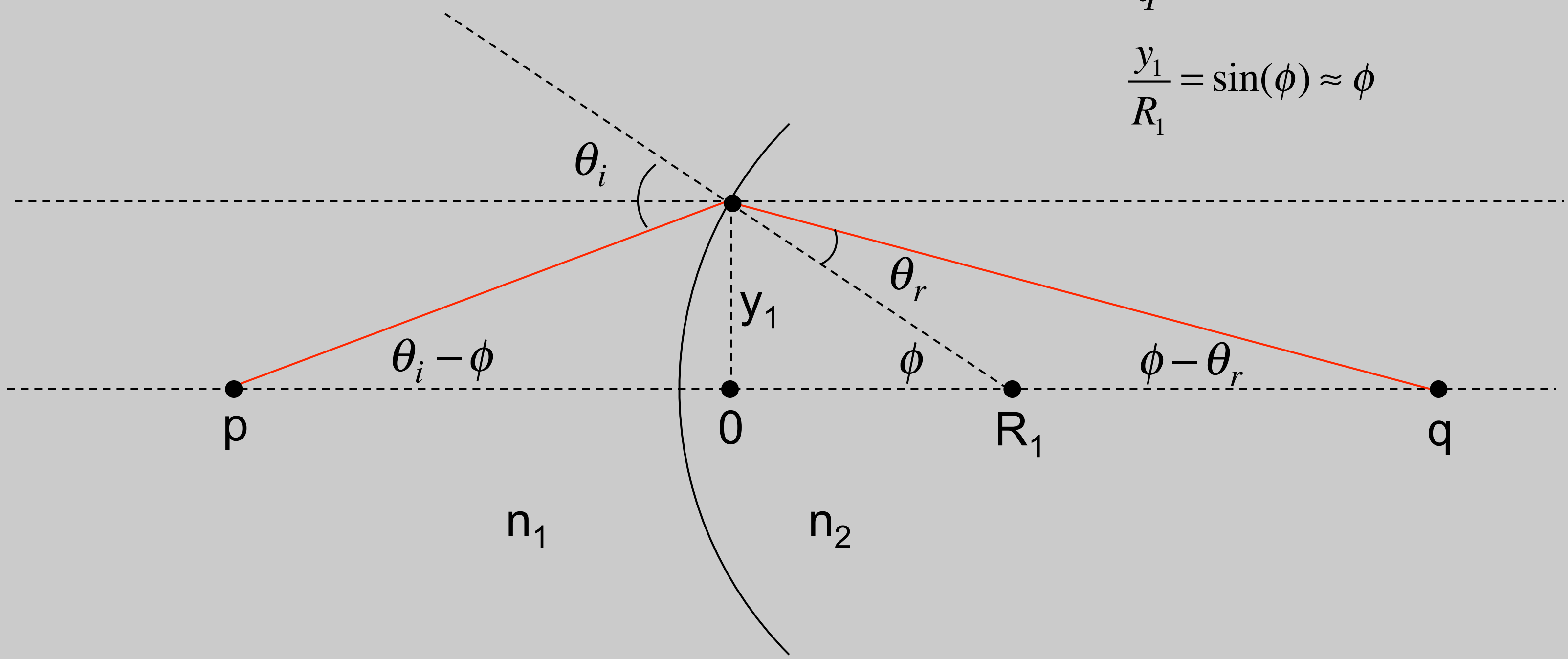
Raytracing: single curved interface

Snell: $n_1 \sin(\theta_i) = n_2 \sin(\theta_r)$

$$\frac{y_1}{p} = \tan(\theta_i - \phi) \approx \theta_i - \phi$$

$$\frac{y_1}{q} = \tan(\phi - \theta_r) \approx \phi - \theta_r$$

$$\frac{y_1}{R_1} = \sin(\phi) \approx \phi$$



$$\frac{n_2}{n_1} = \frac{\sin(\theta_i)}{\sin(\theta_r)} \approx \frac{\theta_i}{\theta_r} = \frac{\phi + \frac{y_1}{p}}{\phi - \frac{y_1}{q}} = \frac{\frac{y_1}{R_1} + \frac{y_1}{p}}{\frac{y_1}{R_1} - \frac{y_1}{q}} = \frac{\frac{1}{R_1} + \frac{1}{p}}{\frac{1}{R_1} - \frac{1}{q}}$$

In paraxial appx, y' s cancel

$$n_2\left(\frac{1}{R_1} - \frac{1}{q}\right) = n_1\left(\frac{1}{R_1} + \frac{1}{p}\right) \rightarrow \frac{1}{R_1}(n_2 - n_1) = \frac{n_2}{q} + \frac{n_1}{p}$$

Raytracing: two curved interfaces

- add second interface: R > 0 if center is to right
- assume $y_2 = y_1$

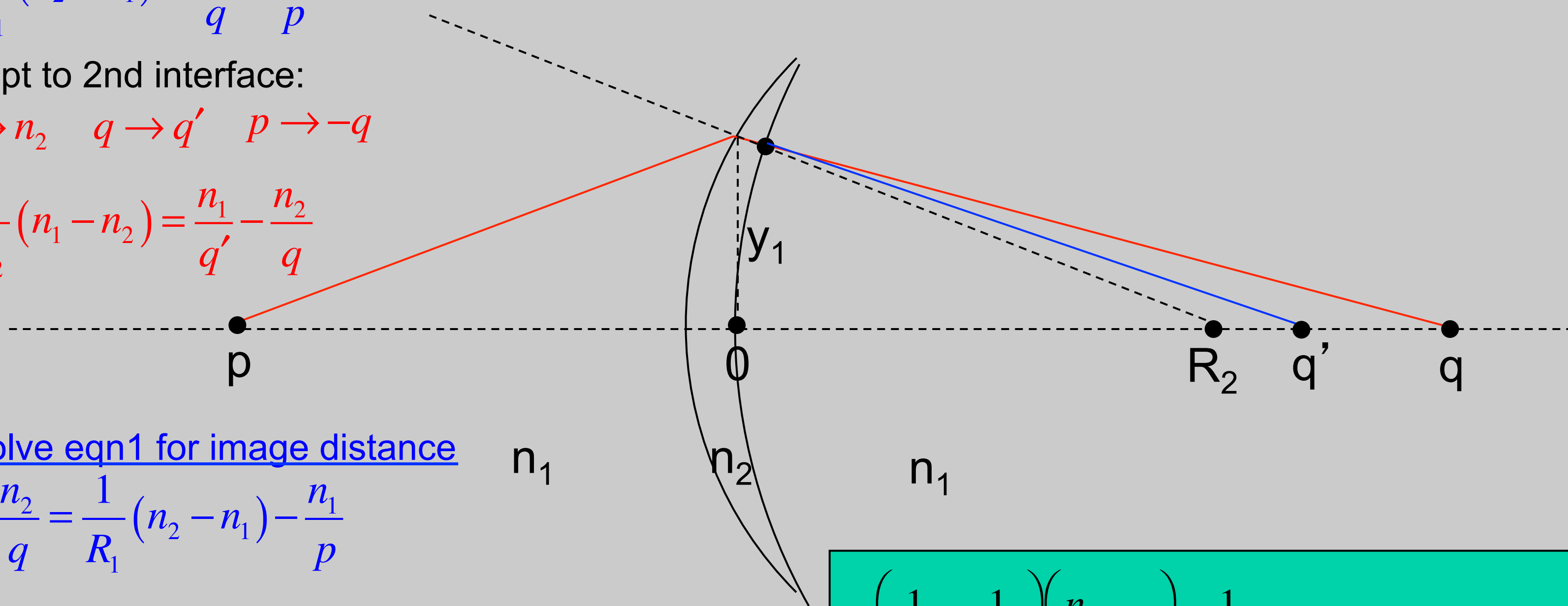
Eqn from 1st:

$$\frac{1}{R_1}(n_2 - n_1) = \frac{n_2}{q} + \frac{n_1}{p}$$

Adapt to 2nd interface:

$$n_1 \leftrightarrow n_2 \quad q \rightarrow q' \quad p \rightarrow -q$$

$$\rightarrow \frac{1}{R_2}(n_1 - n_2) = \frac{n_1}{q'} - \frac{n_2}{q}$$



Solve eqn1 for image distance

$$\rightarrow \frac{n_2}{q} = \frac{1}{R_1}(n_2 - n_1) - \frac{n_1}{p}$$

$$\frac{1}{R_2}(n_1 - n_2) = \frac{n_1}{q'} - \frac{1}{R_1}(n_2 - n_1) + \frac{n_1}{p}$$

$$\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\left(\frac{n_2}{n_1} - 1\right) = \frac{1}{q'} + \frac{1}{p}$$

$$\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\left(\frac{n_2}{n_1} - 1\right) = \frac{1}{f}$$ Focal length (lensmaker's eqn)

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$$ Imaging equation

Raytracing

- Approches:
  - Paraxial tracing (assume small angle to optical axis)
  - Computer tracing (no approximations). Example: Zemax, Oslo,…
- Design procedure
  - Find existing design close to what could work
  - Paraxial trace with ray diagram
    - Calculate magnification, limiting apertures
  - Optimize with ABCD matrices or computer program
  - Analyze aberrations

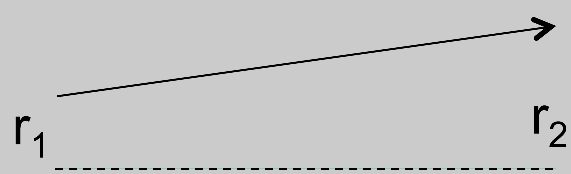
ABCD ray matrices

- Formalism to propagate rays through optical systems
  - Keep track of ray height r and ray angle θ = dr/dz = r'
  - Treat this pair as a vector: $\begin{pmatrix} r \\ r' \end{pmatrix}$
  - Optical system will modify both the ray height and angle, e.g.

$$\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$$

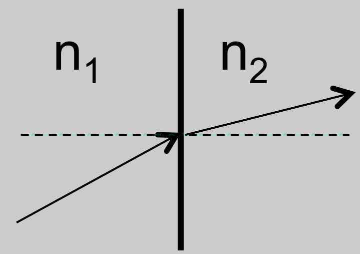
  - Successive ABCD matrices multiply from the left
- Translation

$$\begin{aligned} r_2 &= r_1 + L r_1' \\ r_2' &= r_1' \end{aligned} \quad \rightarrow \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$



Refraction in ABCD

- Translation: $\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$
- Flat interface



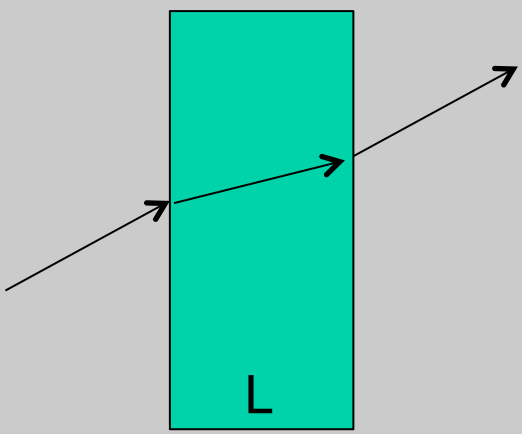
$$r_2 = r_1 \qquad n_1 \sin\theta_1 = n_2 \sin\theta_2 \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$$

$$n_1\theta_1 \approx n_2\theta_2$$

$$r_2' = \frac{n_1}{n_2} r_1'$$

Special case:
$n_1 = 1$, $n_2 = n$ $\rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1/n \end{pmatrix}$

- Window: calculate matrix



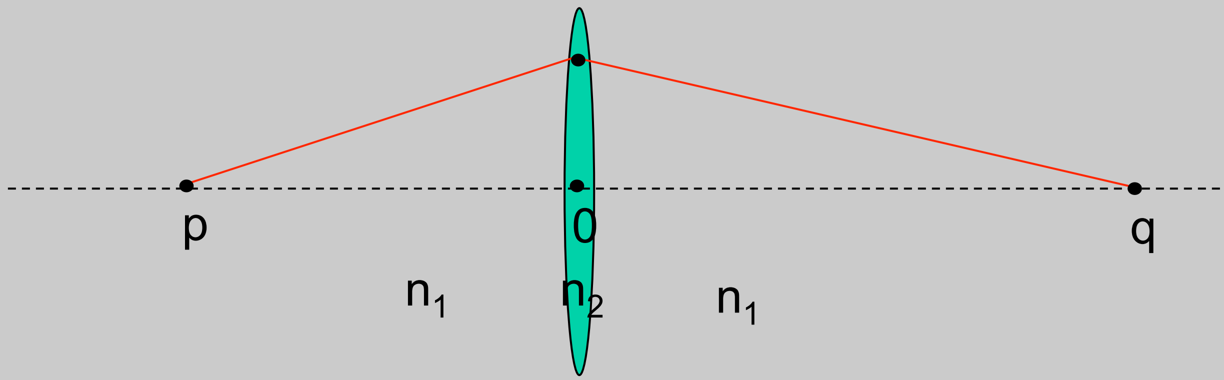
$$\rightarrow \begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1/n \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix} \begin{pmatrix} 1 & L/n \\ 0 & 1/n \end{pmatrix} = \begin{pmatrix} 1 & L/n \\ 0 & 1 \end{pmatrix}$$

Effective thickness reduced by *n*

Curved surfaces in ABCD

- Thin lens: matrix computes transition from one side of lens to other



$$r_2 = r_1$$

$$r_1' = r_1 / p$$

$$r_2' = -r_1 / q$$

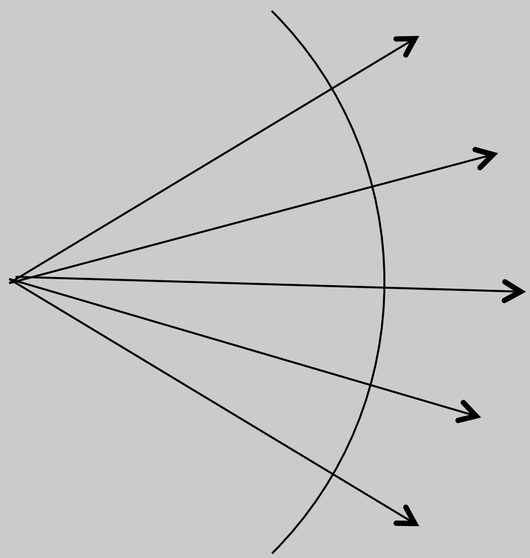
$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} \quad \rightarrow r_2' = -r_1\left(\frac{1}{f} - \frac{1}{p}\right) = -\frac{r_1}{f} + r_1' \quad \rightarrow \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

- Spherical interface: radius *R*

$$\rightarrow \begin{pmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_2}\frac{1}{R} & \frac{n_1}{n_2} \end{pmatrix}$$

Curved wavefronts

- Rays are directed normal to surfaces of constant phase
  - These surfaces are the wavefronts
  - Radius of curvature is approximately at the focal point



- Spherical waves are solutions to the wave equation (away from r = 0)

$$\nabla^2 E + \frac{n^2\omega^2}{c^2} E = 0$$

$$E \propto \frac{1}{r} e^{i(\pm kr - \omega t)}$$

$$I \propto \frac{1}{r^2}$$

Scalar r
+ outward
- inward

Paraxial approximations

- For **rays**, paraxial = small angle to optical axis
  - Ray slope: $\tan\theta \approx \theta$
- For **spherical waves** where power is directed forward:

$$e^{ikr} = \exp\left[ik\sqrt{x^2+y^2+z^2}\right]$$

$$k\sqrt{x^2+y^2+z^2} = kz\sqrt{1+\frac{x^2+y^2}{z^2}} \approx kz\left(1+\frac{x^2+y^2}{2z^2}\right)$$

Expanding to 1st order

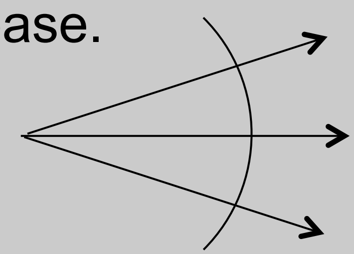
$$e^{i(kr-\omega t)} \to e^{ikz}\exp\left[i\left(k\frac{x^2+y^2}{2z}-\omega t\right)\right]$$

z is radius of curvature

Wavefront = surface of constant phase $k\frac{x^2+y^2}{2z} = \omega t$

For x, y >0, t must increase.

Wave is diverging:



3D wave propagation

$$\nabla^2 \mathbf{E} - \frac{n_j^{\,2}}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = \frac{\partial^2}{\partial z^2}\mathbf{E} + \nabla_\perp^{\;2}\mathbf{E} - \frac{n(\mathbf{r})^2}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = 0$$

$$\nabla_\perp^2 = \partial_x^2 + \partial_y^2 \qquad \boxed{\nabla_\perp^2 = \frac{1}{r}\partial_r\left(r\,\partial_r\right) + \frac{1}{r^2}\partial_\phi^2}$$

- Note:
  - All linear propagation effects are included in LHS: diffraction, interference, focusing…
  - Previously, we assumed plane waves where transverse derivatives are zero.
- More general examples:
  - Gaussian beams (including high-order)
  - Waveguides
  - Arbitrary propagation
  - Can determine discrete solutions to linear equation (e.g. Gaussian modes, waveguide modes), then express fields in terms of those solutions.

Paraxial, slowly-varying approximations

- Assume
  - waves are forward-propagating:

    $$\mathbf{E}(\mathbf{r},t) = \mathbf{A}(\mathbf{r})e^{i(kz-\omega_0 t)} + \text{c.c.}$$

  - Refractive index is isotropic

$$\frac{\partial^2}{\partial z^2}\mathbf{A} + 2ik\frac{\partial}{\partial z}\mathbf{A} - k^2\mathbf{A} + \nabla_\perp{}^2\mathbf{A} + \frac{n^2\omega_0{}^2}{c^2}\mathbf{A} = 0$$

  - Fast oscillating carrier terms cancel (blue)
- Slowly-varying envelope: compare red terms
  - Drop 2nd order deriv if $\frac{2\pi}{\lambda}\frac{1}{L}A \gg \frac{1}{L^2}A$
  - This ignores:
    - Changes in z as fast as the wavlength
    - Counterpropagating waves