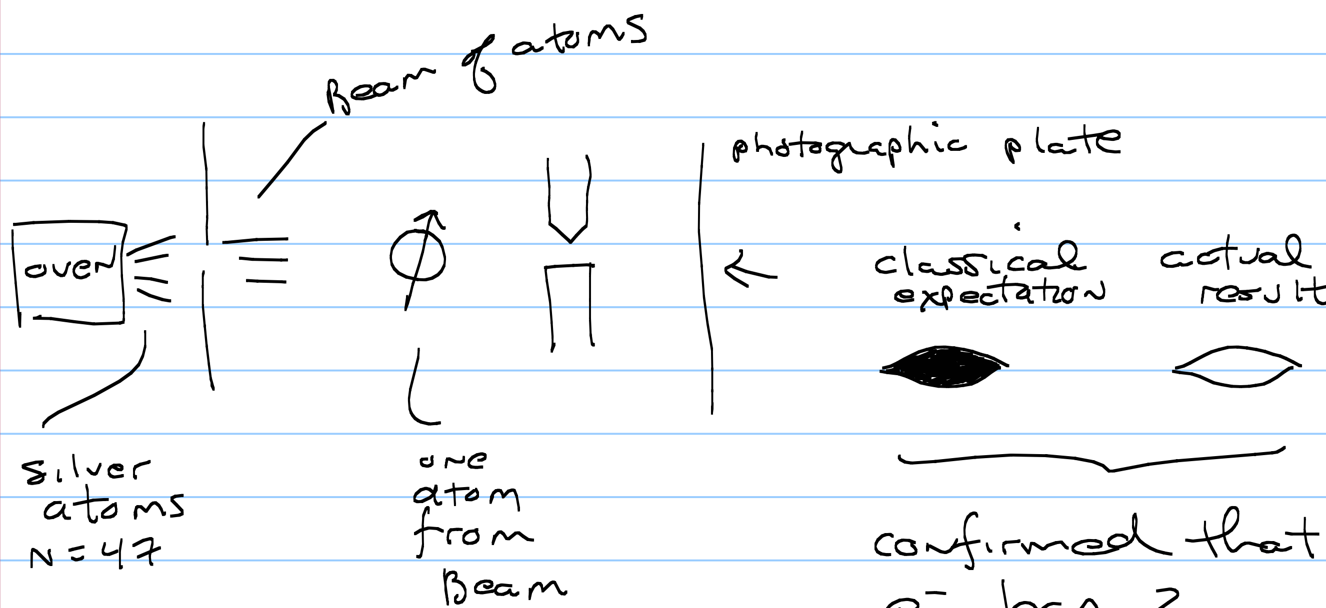
4_23_08

Note Title 4/25/2008

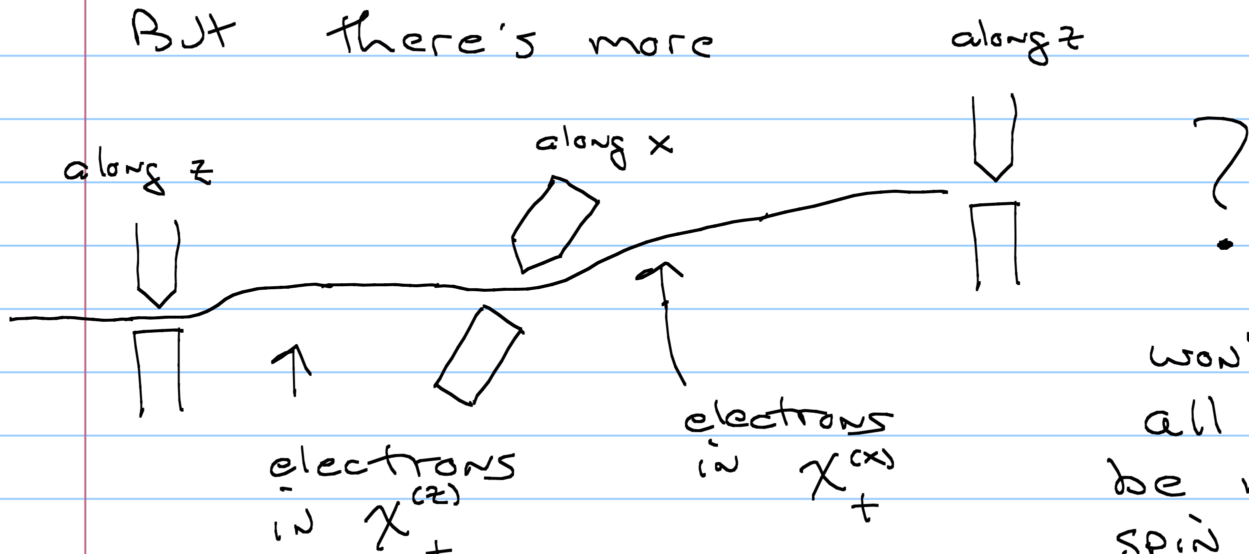
Stern Gerlach

motivation: try to understand atomic origins of magnetism



confirmed that e^- has 2 orientations.

But there's more

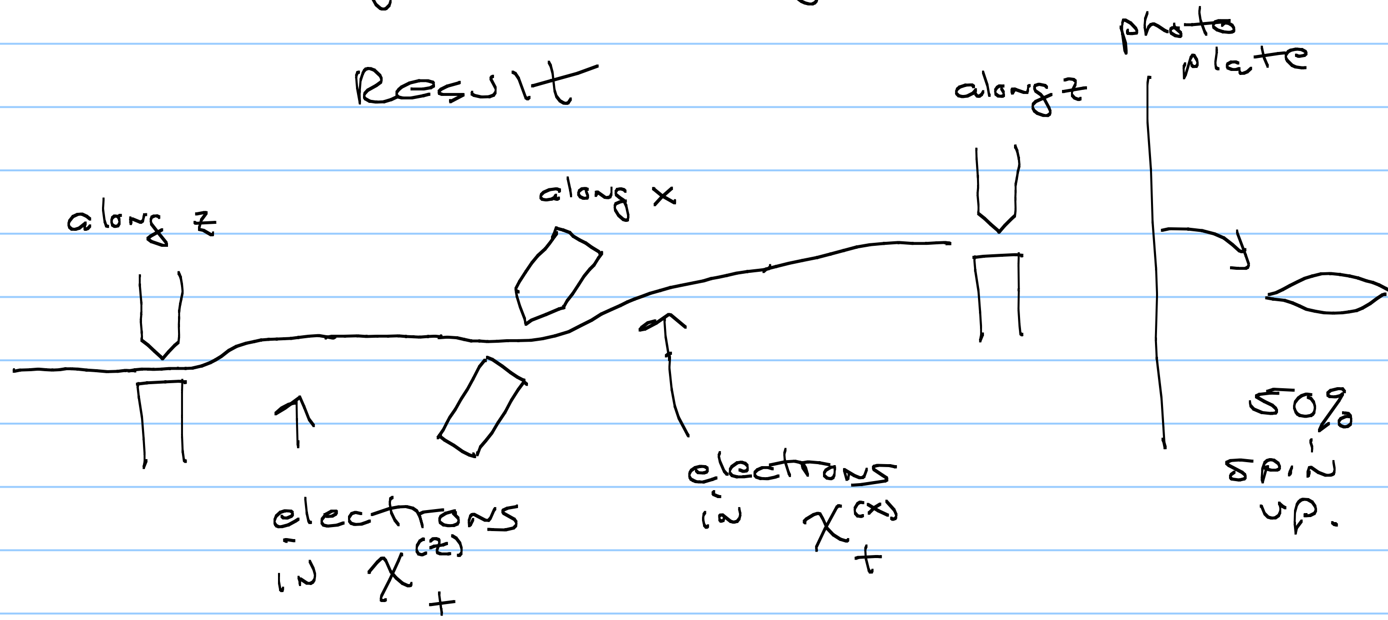


won't these all already be in a spin up (z) state?

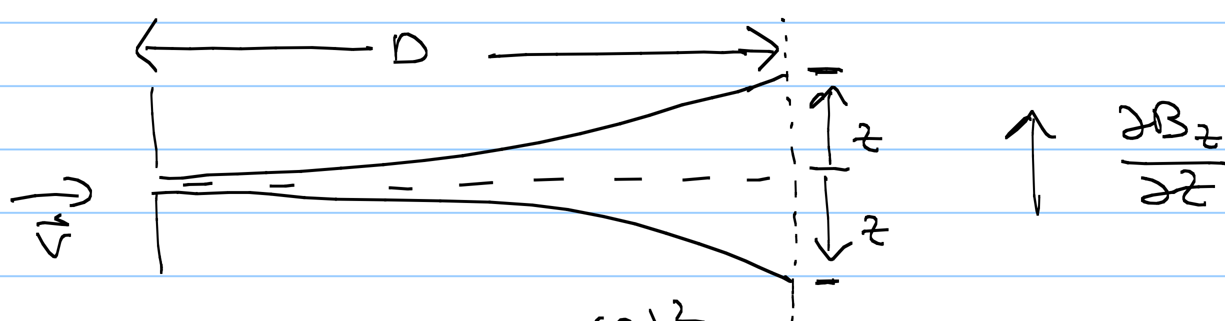
Because $[L_x, L_z] \neq 0$ those observables are incompatible. Thus by measuring L_x we loose precise information about L_z.

So measurement is not a passive selection process it actively projects χ onto an eigenstate of L.

Result



A Detail



$$z = \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{m_e}\left(\frac{D}{v}\right)^2$$

$$= \frac{\left(\frac{1}{2}\right)^2 F D^2}{\frac{1}{2} m_e v^2}$$

Kinetic energy of e^- depends on oven temp.

$$\vec{F} = -\nabla E$$

$$= \nabla(\vec{\mu} \cdot B)$$

$$= \pm \mu \frac{\partial B_z}{\partial z}$$

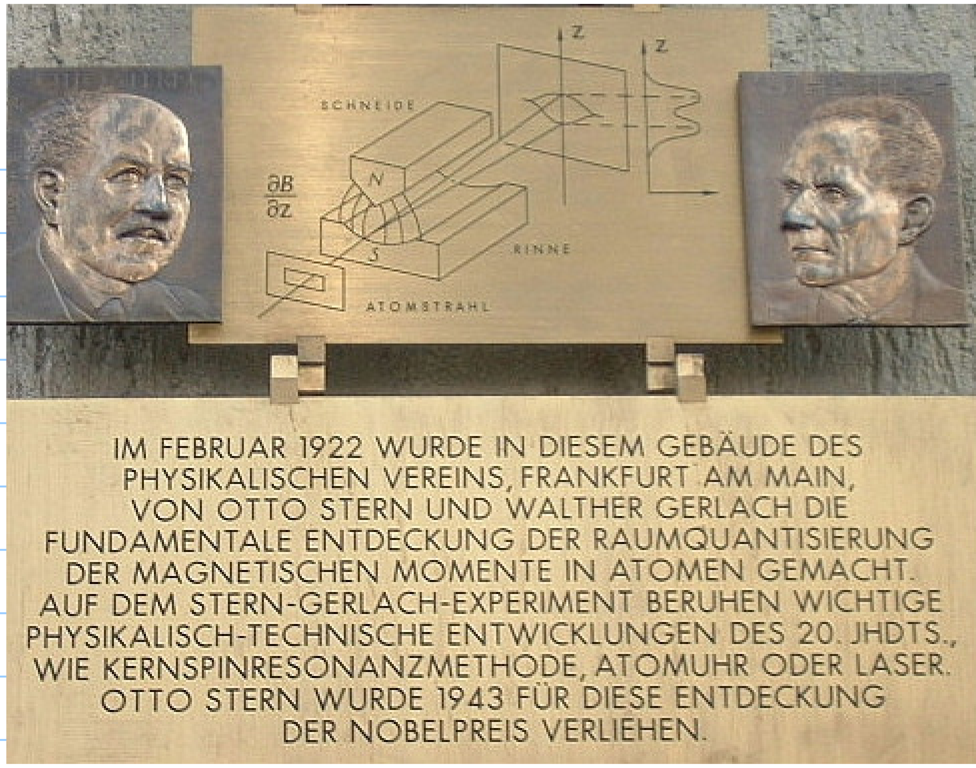
depending on spin state

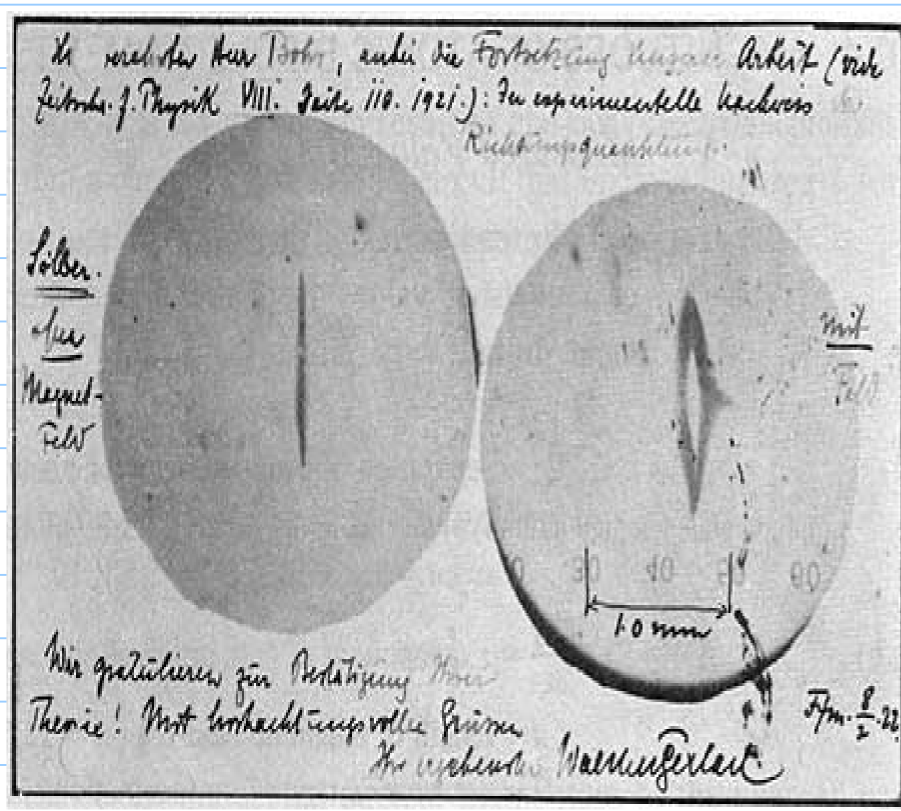
So $z = \pm \dfrac{D^2}{4\,K.E.} \mu \dfrac{\partial B}{\partial z}$

by measuring z we can estimate μ.

This msmt was done in 1921

It was only in 1925 that Goudsmit & Uhlenbeck postulated that the e^- had an intrinsic angular momentum independent of its orbital characteristics





Postcard from Gerlach to Bohr
congratulating him on the

success of his theory.

Digression

Consider a superposition of 2 kets

$$|\psi\rangle + e^{i\theta}|\phi\rangle$$

Now, $e^{i\theta}|\phi\rangle$ is the same state as $|\phi\rangle$ since all measurements will yield the same results.

However $|\psi\rangle + e^{i\theta}|\phi\rangle$ is not the same as $|\psi\rangle + |\phi\rangle$ since

$$(|\psi\rangle + e^{i\theta}|\phi\rangle)^{+}(|\psi\rangle + e^{i\theta}|\phi\rangle)$$

$$= \left(\langle\psi| + e^{-i\theta}\langle\phi|\right)\left(|\psi\rangle + e^{i\theta}|\phi\rangle\right)$$

$$= \langle\psi|\psi\rangle + \langle\phi|\phi\rangle + e^{-i\theta}\langle\phi|\psi\rangle + e^{i\theta}\langle\psi|\phi\rangle$$

$$= \langle\psi|\psi\rangle + \langle\phi|\phi\rangle + \underbrace{e^{-i\theta}\langle\phi|\psi\rangle + e^{i\theta}\langle\phi|\psi\rangle^{*}}_{2\,\mathrm{Re}\left(e^{-i\theta}\langle\phi|\psi\rangle\right)}$$

interference term

whereas $e^{i\theta}(|\psi\rangle + |\phi\rangle) \Rightarrow$

$e^{-i\theta}(\langle\psi| + \langle\phi|)\, e^{i\theta}(|\psi\rangle + |\phi\rangle)$

since $e^{-i\theta}$, $e^{i\theta}$ are numbers

$= \langle\psi|\psi\rangle + \langle\phi|\phi\rangle + 2\mathrm{Re}\langle\phi|\psi\rangle$

So, a relative phase matters, an overall phase does not.

Suppose we have 2 spin-$\frac{1}{2}$ particles. E.g. electron + proton in ground state of H. (so $l=0$ for simplicity). χ_1 + χ_2 the spinors associated with each particle

Since each can have spin up or down there are 4 possibilities

$\uparrow\uparrow \quad \uparrow\downarrow \quad \downarrow\uparrow \quad \downarrow\downarrow$

Let $\vec{S} = \vec{S}^{(1)} + \vec{S}^{(2)}$

so that $S_z = S_z^{(1)} + S_z^{(2)}$.

Assume $S_z^{(1)}$ acts only on χ_1

$S_z^{(2)}$ " " χ_2

Then $S_z \chi_1 \chi_2 =$

$$\left(S_z^{(1)} + S_z^{(2)}\right) \chi_1 \chi_2 = \left(S_z^{(1)} \chi_1\right) \chi_2 + \chi_1 \left(S_z^{(2)} \chi_2\right)$$

$$= \hbar m_1 \chi_1 \chi_2 + \hbar m_2 \chi_1 \chi_2$$
$$= \hbar (m_1 + m_2) \chi_1 \chi_2$$

m_1	m_2	$m_1 + m_2$	
$-1/2$	$-1/2$	-1	$\downarrow \downarrow$
$-1/2$	$1/2$	0	$\downarrow \uparrow$
$1/2$	$-1/2$	0	$\uparrow \downarrow$
$1/2$	$1/2$	1	$\uparrow \uparrow$

our composit system is described by the ket
$|s\ m\rangle$
For $s = 1$, $m = -1, 0, 1$.

But why is there an "extra" $m=0$ State? Because $m=0$ is a superposition of $\uparrow\downarrow$ and $\downarrow\uparrow$.

$$S_- (\uparrow\uparrow) = \left(S_-^{(1)} + S_-^{(2)}\right)(\uparrow\uparrow)$$

$$= \left(S_-^{(1)} \uparrow\right)\uparrow + \uparrow S_-^{(2)} \uparrow$$
$$= \hbar \downarrow\uparrow + \hbar \uparrow\downarrow$$

So the $s=1$ triplet of ~~states~~ is

$$\left.\begin{aligned} |1\ 1\rangle &= \uparrow\uparrow \\ |1\ 0\rangle &= \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\ |1\ {-1}\rangle &= \downarrow\downarrow \end{aligned}\right\} \quad s=1 \text{ triplet}$$

For $s=0$ $\quad m=0$

$$|0\ 0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \qquad \overset{s=0}{\text{singlet}}$$