Phys 361 Homework 7

1) Let's think a bit more about nonlinear optics. That is, situations in which the polarization of some material is not a strictly linear function of the electric field. You may have to dredge up some of what we learned about optics in Phys 200, or even look something up.

a) Instead of $P = \chi\varepsilon_0 E$ being the relationship between polarization and field, let's expand it as $P = \chi_1\varepsilon_0 E + \chi_2\varepsilon_0 E^2 + \chi_3\varepsilon_0 E^3$. We're basically saying the polarization is a series expansion, and keeping terms beyond the linear term. Now assume an electromagnetic wave is incident on a crystal whose polarization relationship looks like this. As you may recall from Phys 200, we often write the time-dependent part of a plane wave as $E(t) = E_0 \sin \omega t$, where ω is the angular frequency of the wave. Our crystal is going to re-radiate according to the polarization it experiences, so plug that E into the expansion for P and show mathematically that we can expect outgoing waves with higher frequencies to show up.

b) If I shoot an infrared laser with wavelength 1064 nm at this crystal, what will be the wavelengths of the second and third harmonics?

c) There exist several ways to create laser light in the violet/blue range (400 to 480 nm). Some involve nonlinear optics and some don't. Root around on the web and see what you can find out about violet and blue lasers and report on what you find (be specific about things like the wavelengths involved). See if you can find at least one instance involving nonlinear optics.

2) Let's suppose you have a chunk of neutral dielectric material. You heat it up, apply an external electric field to polarize it, and cool it down while the field is in place. Then you take the field away. In this fashion, you can manufacture an object that stays polarized even though there may not be any external fields or charges in the neighborhood (such an object is called an electret and is an actual thing).

Let's further suppose that we have a finished electret that is spherical in shape with radius *a* and has been given a polarization throughout of the form:

$$\vec{P} = kr\hat{r}$$

Where *k* is an arbitrary constant. Note that we haven't added or removed any net charge from it, and the source of the electric field that did the polarizing is long gone. There's nothing present except the polarized sphere.

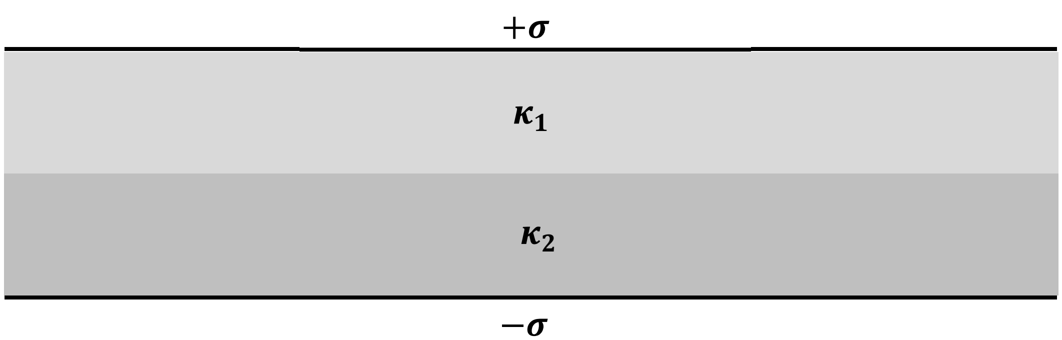
a) Find the bound volume charge density ρ_b in this sphere and the bound surface charge σ_b. Explicitly calculate the total bound volume charge and the total bound surface charge and compare them. Is the result sensible?

b) Calculate the electric field **E** inside and outside the sphere. Also calculate the electric displacement field **D** inside and outside the sphere.

c) Give me a sketch and a qualitative description of how the charges in the sphere must be arranged to produce what we have here.

3) (based on Pollack and Stump 6.14)

We have two isolated, square, parallel conducting plates of side length L and separation d ($L \gg d$). We charge them with some $+\sigma$ on the top plate and $-\sigma$ on the bottom plate. Now we fill up the space in between with two slabs of dielectric material, each of thickness $d/2$, with dielectric constants κ_1 and κ_2. The system is shown below:



a) Find $\vec{D}$ everywhere between the plates.

b) Find $\vec{E}$ everywhere between the plates.

c) Find the bound surface charge densities on each of the three dielectric surfaces, and find the bound volume charge density in each dielectric.

d) Show that the capacitance of the system is given by:

$$C = \frac{2\varepsilon_0 L^2 \kappa_1 \kappa_2}{d(\kappa_1 + \kappa_2)}$$

Note: The work we do here is basically the derivation of one of the rules we learned in Phys 200 for making capacitors that are filled with different combinations of dielectrics. And that means you don't get to just use that series addition rule for part d.

4) (based on Pollack and Stump 6.17)

Consider an electric field line passing through a planar interface between two insulating media with dielectric constants κ_1 and κ_2. Assume there is no free charge on the interface. Let θ_1 and θ_2 be the angles between the field line and the normal to the interface in the two regions. Prove that $\kappa_1 \cot\theta_1 = \kappa_2 \cot\theta_2$

Note that this problem is basically some practice with applying our boundary conditions in matter. As part of your answer, include a diagram of the electric field lines in the two regions and the angles they make. Specify which region has the larger κ, and make your diagram consistent with that. The form of the equation we're proving and the diagram you're making should be very reminiscent of Snell's law. Coincidence? We'll find out when we do electromagnetic waves and derive Snell's law from first principles.