Conservation of energy in differential form.

informational: What is the infinitesimal work done by the electromagnetic fields on a charge?

$$dw = \vec{F}_{EM} \cdot d\vec{r}$$

$$dw = \left(q\vec{E} \times q\vec{v} \times \vec{B}\right) \cdot d\vec{r} = \left(q\vec{E} + q\vec{v} \times \vec{B}\right) \cdot \vec{v}\, dt$$

informational: What is the power delivered to the charge by the electromagnetic fields?

$$\frac{dw}{dt} = q\vec{E} \cdot \vec{v}$$

congruous: How is this modified if we have more than one charge?

$$\frac{dw}{dt} = \rho\, d\tau\, \vec{E} \cdot \vec{v} = \vec{E} \cdot \vec{J}\, d\tau$$

$$\vec{J}$$

$$W_{net} = \Delta KE$$

$$W_{noncons} + W_{cons} = \Delta KE$$

$$W_{cons} \equiv -\Delta PE$$

$$W_{noncons} = \Delta(KE + PE)$$

informational: Which of these 3 parameters (W, KE, PE) did we calculate above?

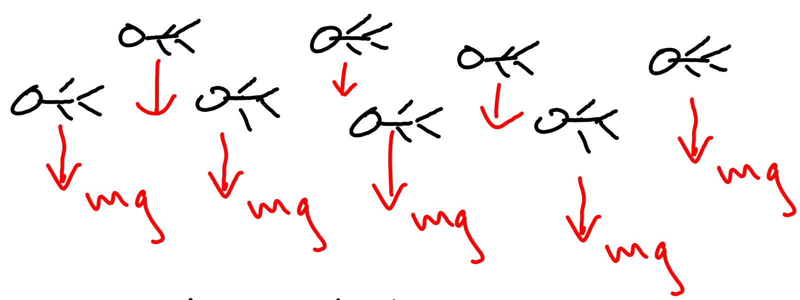
analogy: How does the above calculation relate to the skydiver problem?

Now assume no friction.

$$dW_{gravity} = \vec{F}\cdot d\vec{r} = m\vec{g}\cdot\vec{v}\,dt$$

$$\frac{dW_{gravity}}{dt} = m\vec{g}\cdot\vec{v}$$

congruous: How is this modified if we have more than one skydiver?



$$\frac{dW_{gravity}}{dt} = \rho\, d\tau\, \vec{g}\cdot\vec{v} = \vec{g}\cdot\vec{J}\,d\tau$$

$$\vec{J} = \rho v$$

For homework show

$$\vec{E}\cdot\vec{J} = \frac{1}{\mu_0}\vec{E}\cdot(\vec{\nabla}\times\vec{B}) - \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

For homework show

$$\vec{E}\cdot(\vec{\nabla}\times\vec{B}) = -\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} - \vec{\nabla}\cdot(\vec{E}\times\vec{B})$$

Look up and use the vector identity for $\vec{\nabla}\cdot(\vec{E}\times\vec{B})$.

For homework use

$$\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(B^2) \quad \& \quad \vec{E}\cdot\frac{\partial\vec{E}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(E^2)$$

to show that

$$\vec{E}\cdot\vec{J} = -\frac{1}{2}\frac{\partial}{\partial t}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \frac{1}{\mu_0}\vec{\nabla}\cdot(\vec{E}\times\vec{B})$$

For homework apply the divergence theorem to show that

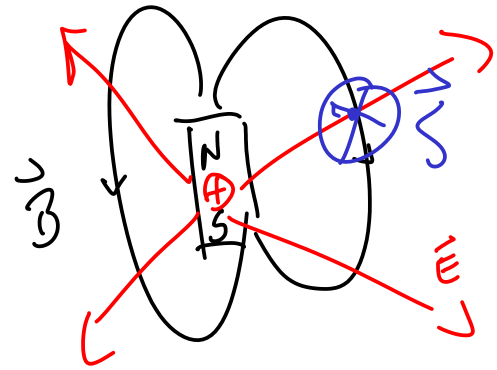
$$\frac{dW}{dt} = -\frac{d}{dt}\underbrace{\int\frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)d\tau}_{U_{EM}} - \frac{1}{\mu_0}\oint\underbrace{(\vec{E}\times\vec{B})}_{\vec{S}}\cdot d\vec{a}$$

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ is Poynting vector

Conservation of electromagnetic energy is then

$$\frac{dW_{EM}}{dt} = -\frac{d}{dt}U_{EM} - \oint\vec{S}\cdot d\vec{a}$$

Questions:



incongruous: how there be energy flowing
around this configuration of E and B?

analogy: how is this related to conservation of charge or conservation of energy
in heat flow?

congruous: It is 1885 and your name is Poynting. You have just derived this result and
want to see if it makes sense. How are you going to do that?

congruous: This is only a part of the work energy theorem dealing with conservative
forces. How does this fit into the total work energy theorem.

more

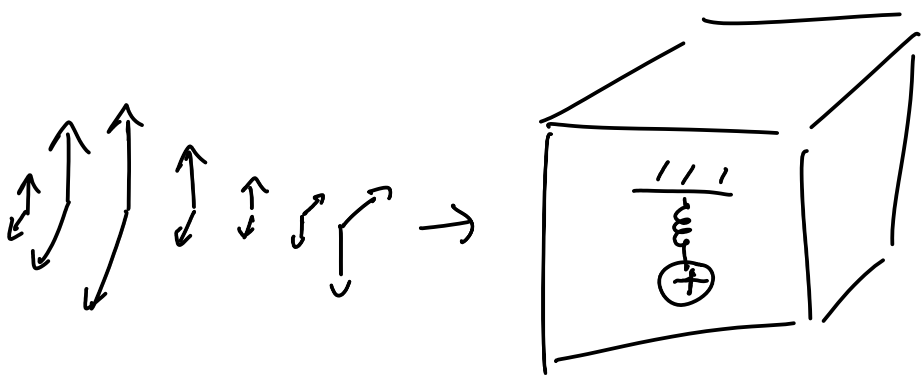
analogy: how is this related to conservation of charge or conservation of energy in heat flow?

$$\frac{dW_{EM}}{dt} = -\frac{d}{dt}U_{EM} - \oint \vec{S}\cdot d\vec{a}$$

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t} \xrightarrow[\text{theorem}]{\text{divergence}} \oint \vec{J}\cdot d\vec{a} = -\frac{d}{dt}Q_{enclosed}$$

The flow of charge out of a closed surface is equal to the change in total charge within that surface.

There is no third term as in our Poynting theorem. So what does $\frac{dW_{EM}}{dt}$ mean?



There is EM energy in the box given by $\int \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)d\tau$

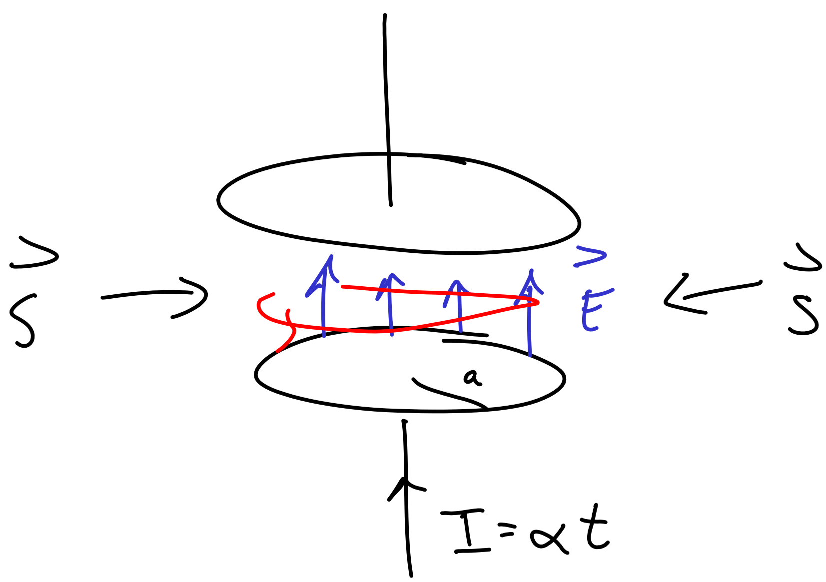
The time derivative of this gives the rate of change of energy in the box.

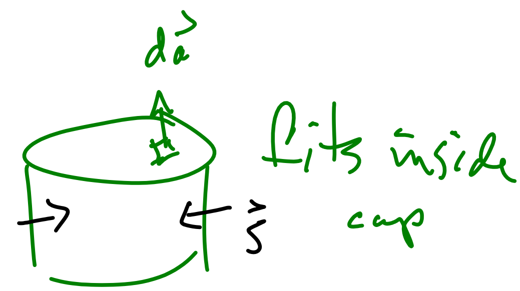
There is a flow of energy into or out of the box given by $\frac{1}{\mu_0}\oint(\vec{E}\times\vec{B})\cdot d\vec{a}$

The difference between these time rates of change in energy is due to a source or sink of energy. The charges can be given potential energy or the potential energy of the charges can change to generate electromagnetic fields.

The source or sink term is $\dfrac{dW_{EM}}{dt}$

congruous: It is 1885 and your name is Poynting. You have just derived this result and want to see if it makes sense. How are you going to do that?





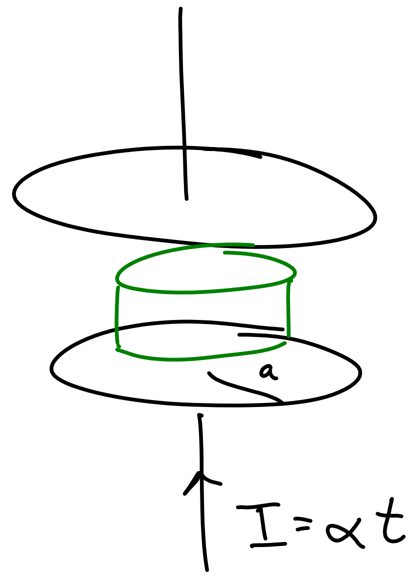
$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 \pi a^2}\,; \quad Q = \alpha t \Rightarrow E = \frac{\alpha t}{\pi \epsilon_0 a^2}$$

$$B\, 2\pi r = \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \pi r^2\,; \quad B = \mu_0 \frac{\alpha r}{\pi a^2}$$

$$u_{em} = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) =$$

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right) =$$

$$\frac{\partial u_{em}}{\partial t} =$$



$$\vec{\nabla} \cdot \vec{S} =$$

in cylindrical coords

$$\oint \vec{S} \cdot d\vec{a}$$

on cylindrical surface

$$\frac{dW_{EM}}{dt} = -\frac{d}{dt} U_{EM} - \oint \vec{S} \cdot d\vec{a}$$

$$\parallel$$

$$0$$

Since there are no charges within the surface electromagnetic energy can't be sourced or sunk within the surface.

congruous: This is only a part of the work energy theorem dealing with conservative forces. How does this fit into the total work energy theorem.

$$\frac{dW_{EM}}{dt} = \underbrace{\rho\, d\tau\, \vec{E}\cdot\vec{v}}_{\vec{J}} = \vec{E}\cdot\vec{J}\, d\tau$$

$$W_{net} = \Delta KE$$

$$W_{non\,cons} + \underbrace{W_{cons}}_{= W_{EM}} = \Delta KE$$

$$W_{EM} = \Delta KE - W_{non\text{-}cons}$$

$$\frac{dW_{EM}}{dt} = \frac{d}{dt}KE - \frac{d}{dt}W_{non\text{-}cons} = \frac{d}{dt}U_{KE} - \frac{d}{dt}U_{nc}$$

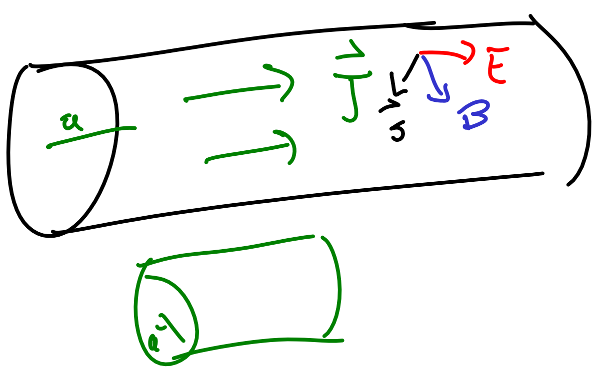
$$U_{KE} = \int u_{KE}\, d\tau$$

If there is no change in KE as in an Ohmic material then $U_{KE} = 0$

$$\frac{dW_{EM}}{dt} = \text{Power}_{resistor} = -\frac{d}{dt}U_{EM} - \oint \vec{S}\cdot d\vec{a}$$

congruous: how would I set up a simple problem to show this?

copper wire



$$\vec{J} = \sigma \vec{E}$$

congruous: How do I calculate the Poynting vector?

congruous; How does the electromagnetic energy within the surface change with time?

There is no change of electromagnetic energy within the surface so

$$P_{resistor} = \oint \vec{S} \cdot d\vec{a} = I^2 R$$