

(b) We need to settle  $C_1$ ,  $C_2$ , and  $\gamma$  in our  $B(y)$ .

We know that  $B_{1z} = B_{2z}$  at  $y=0$  and  $y=b$ , but that doesn't help much since  $B_{1z} = 0$  always.

What about  $E_{11}$ ? Since  $E=0$  in the conductor, the continuity of  $E_{11}$  means the  $k$  component of  $E$  evaluated at  $y=0$  or  $y=b$  must give 0. Therefore:

$$B'(0) = B'(b) = 0 \Rightarrow \gamma C_1 \cos(0) - \gamma C_2 \sin(0) = 0$$

$$\Rightarrow C_1 = 0$$

Evaluated at  $b$ :  $-\gamma C_2 \sin(\gamma b) = 0$

$$\Rightarrow \gamma b = n\pi, \text{ integer } n$$

$$\Rightarrow \gamma = \frac{n\pi}{b}$$

Which allows us to get a complete  $B(y)$  of  $B_0 \cos(\frac{n\pi y}{b})$ , full  $E+B$  fields, and the dispersion relation:

$$\vec{B} = B_0 \cos\left(\frac{n\pi y}{b}\right) e^{i(kz - \omega t)} \hat{i}$$

$$\vec{E} = -\frac{c^2 k}{\omega} B_0 \cos\left(\frac{n\pi y}{b}\right) e^{i(kz - \omega t)} \hat{j} + \frac{n\pi c^2}{ib\omega} B_0 \sin\left(\frac{n\pi y}{b}\right) e^{i(kz - \omega t)} \hat{k}$$

$$\frac{n\pi}{b} = \sqrt{\omega^2/c^2 - k^2} \quad (\text{taking the real parts, of course})$$

Next we use the other two boundary conditions to find  $\sigma + \vec{K}$ :

$$E_{1z} - E_{2z} = \sigma/\epsilon_0 \quad (E_{2z} = 0) \text{ gives (at } y=0 \text{ or } y=b)$$

$$-\frac{c^2 k}{\omega} B_0 \cos\left[\frac{n\pi}{b}(b \text{ or } 0)\right] e^{i(kz - \omega t)} = \sigma/\epsilon_0 \quad \text{cos}$$

$$\Rightarrow \sigma = -\frac{c^2 k \epsilon_0}{\omega} B_0 \cos(kz - \omega t) \quad \text{at } y=0$$

$$= -\frac{c^2 k \epsilon_0}{\omega} B_0 \cos(n\pi) \cos(kz - \omega t) \quad \text{at } y=b \quad \left( \begin{array}{l} \cos(n\pi) = 1 \text{ for} \\ \text{even } n, \quad -1 \text{ for odd} \end{array} \right)$$

Similarly,  $\vec{B}_{11} - \vec{B}_{21} = \mu_0 \vec{K} \times \hat{n}$  gives our surface current:

$$B_0 \cos(kz - \omega t) \hat{i} = \mu_0 \vec{K} \times \hat{j} \quad (\text{for } y=0)$$

$$\Rightarrow \vec{K} = -\frac{B_0}{\mu_0} \cos(kz - \omega t) \hat{k}$$

Also  $B_0 \cos(n\pi) \cos(kz - \omega t) \hat{i} = \mu_0 \vec{K} \times (-\hat{j})$  (for  $y=b$ , note  $\hat{n}$  becomes  $-\hat{j}$  instead of  $\hat{j}$ )

$$\Rightarrow \vec{K} = +\frac{B_0}{\mu_0} \cos(kz - \omega t) \hat{k} \quad \text{if } n \text{ is even}$$

$$- \frac{B_0}{\mu_0} \cos(kz - \omega t) \hat{k} \quad \text{if } n \text{ is odd}$$