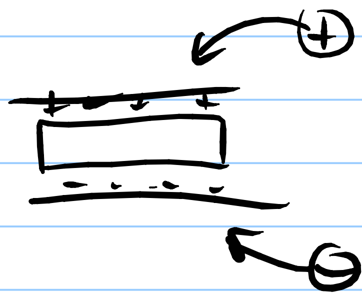
Energy in Dielectrics

Note Title 2/28/2011



free charge

const speed

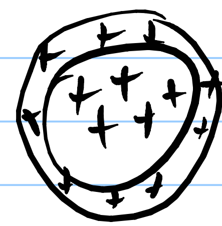
$$W_{nc} = W_{me} = \Delta(\cancel{KE} + PE)$$

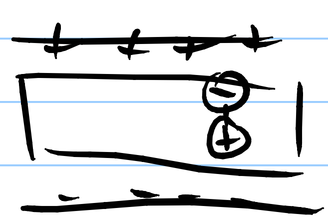
0

$$\Delta PE = \Delta(qV)$$

$$dW_{me} \to dq\, V(q)$$

$$\Delta W_{me} = \int \Delta\rho_f V(r)\, d\tau \qquad \rho\, d\tau = dq$$



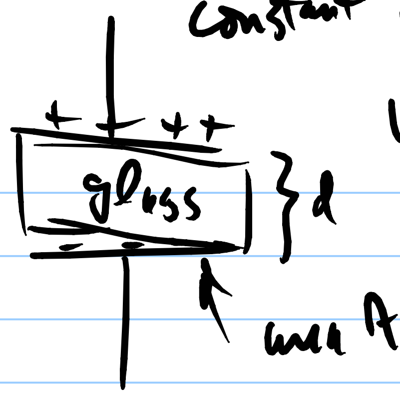


$$\vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\Delta W = \Delta\left(\frac{1}{2}\int \vec{D}\cdot\vec{E}\, d\tau\right)$$

$$\int \frac{1}{2}\epsilon_0 E^2\, d\tau$$

constant Q



$$W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{\sigma_{vac}^2 A^2}{\epsilon \frac{A}{d}}$$

$$W = \frac{1}{2}\frac{\sigma_{vac}^2 A d}{\epsilon_0(1+\chi_e)}$$

$$\int \underbrace{\frac{1}{2}\vec{D}\cdot\vec{E}}_{\text{const in cap}}\, d\tau \qquad D = \epsilon E = \epsilon_0(1+\chi_e)E$$

$$W = \frac{1}{2}\epsilon_0(1+\chi_e)E^2 A d$$

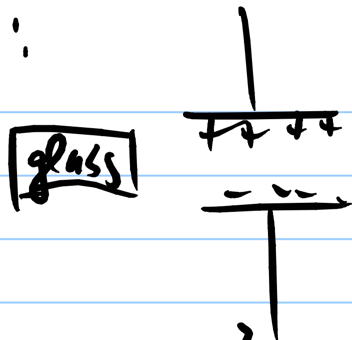
$$= \underbrace{\frac{1}{2}\epsilon_0(1+\chi_e)\left[\sigma^{vac}\frac{1}{\epsilon_0(1+\chi_e)}\right]^2}_{J/m^3}\underbrace{Ad}_{m^3}$$

$$= \frac{1}{2}\cancel{\epsilon_0}\cancel{(1+\chi_e)}\frac{\sigma^{vac^2}}{\epsilon_0^{\cancel{2}}(1+\chi_e)^{\cancel{2}}}Ad = \frac{1}{2}\frac{\sigma^{vac^2}Ad}{\epsilon_0(1+\chi_e)}$$

$$W = \frac{1}{2}\frac{\sigma^{vac^2}Ad}{\epsilon_0(1+\chi_e)}$$

Constant Q

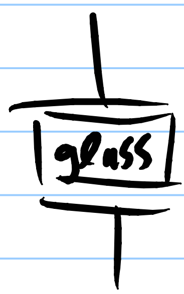
Example:



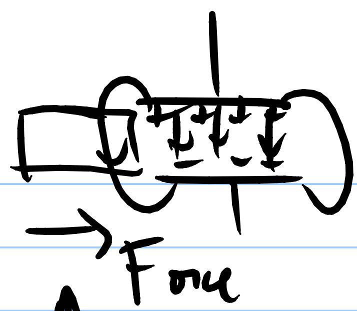
$$dW = dq\,V = dq\,\frac{q}{C}$$

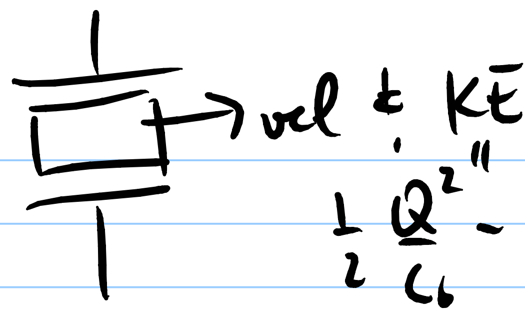
$$\int dW = \frac{1}{2}\frac{Q^2}{C}$$

$\frac{1}{2}\frac{Q^2}{C_0} \leftarrow$ without glass



$$W_{glass} = \frac{1}{2}\frac{Q^2}{KC_0} \qquad K > 1 \qquad W_{glass} < W_{vac}$$

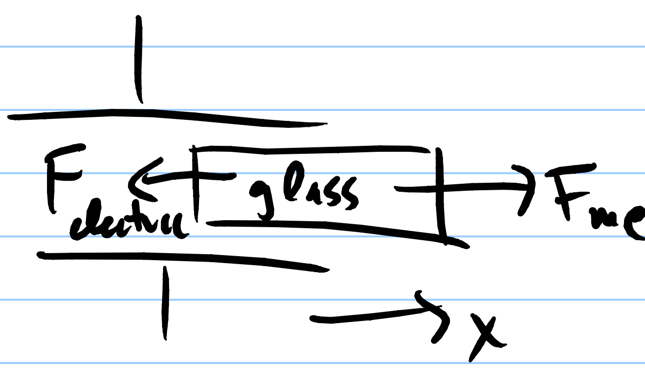




$$\frac{1}{2}\frac{Q^2}{C_0} - \frac{1}{2}\frac{Q^2}{KC_0}$$

complicated

conservation simple



$$W_{nc} = \Delta(\cancel{KE} + PE)$$

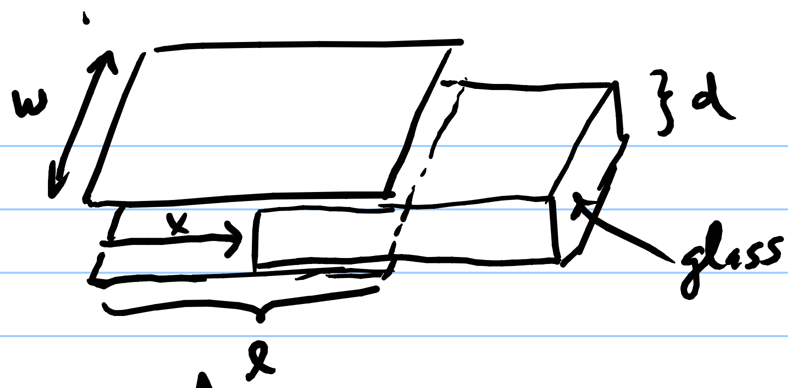
$\cancel{KE} \to 0$ const speed

$W_{nc} = W_{me}$

$\Downarrow$

$$dW = F_{me}\, dx = -F_{electrostatic}\, dx$$

$$F_{electr} = -\frac{dW}{dx} \quad \text{find } W(x) = \frac{1}{2}\frac{Q^2}{C(x)}$$



two caps in parallel

$$C(x) = \frac{\epsilon_0 w}{d}\left(\epsilon - \chi_e x\right)$$

$$F = -\frac{dW}{dx} = +\frac{1}{2}\frac{Q^2}{C^2}\frac{dC(x)}{dx}$$