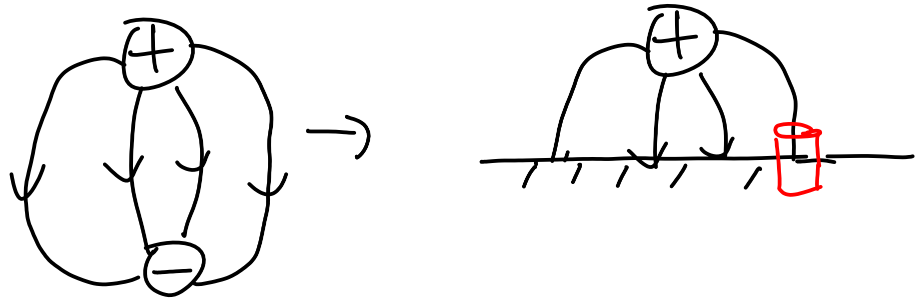
Hmwk soln assign 4

1.) You are given the voltage for two point charges, one above and one below the axis. You are told this is a soln to the charge above and a conducting plane located at z=0. As shown in class using Gauss's law.

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{z}$$



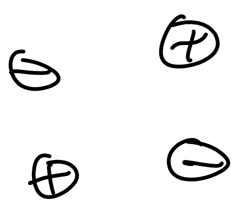
You need to find E near the conductor. $\vec{E} = -\nabla V$

Since E will be perpendicular to the surface you need on find the component.

2.) See the other pdf file posted in this section of the wiki for this soln.

3.) See the other pdf file for this soln.

4.) (a)



$$V = \frac{1}{4\pi\epsilon_0}\sum' \frac{q_i}{r_{ij}} = \frac{1}{4\pi\epsilon_0}\left\{-\frac{q}{a} + \frac{q}{\sqrt{2}a} - \frac{q}{a}\right\}$$

$$W = qV$$

1b) $W_1 = 0$; $W_2 = \frac{1}{4\pi\epsilon_0}\left(-\frac{q^2}{a}\right)$; $W_3 = \frac{1}{4\pi\epsilon_0}\left(\frac{q^2}{\sqrt{2}a} - \frac{q^2}{a}\right)$; W_4

5.) $W = \frac{1}{2}\int \rho V \, d\tau$ $\quad V = -\int \vec{E}\cdot d\vec{r} = -\int^{R} \vec{E}_{out}\cdot d\vec{r} - \int_{R}^{r} \vec{E}_{in}\cdot d\vec{r}$

$$\vec{E}_{out} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$ using Gauss's law

$$\vec{E}_{in} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^3} r\hat{r}$$

for $r<R$ $\quad V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{R^3}\left(\frac{r^2-R^2}{2}\right)\right]$

$$= \frac{q}{4\pi\epsilon_0}\frac{1}{2R}\left(3-\frac{r^2}{R^2}\right)$$

$$W = \frac{1}{2}\rho\frac{1}{4\pi\epsilon_0}\frac{q}{2R}\int^{R}\left(3-\frac{r^2}{R^2}\right)4\pi r^2\,dr$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\hat{r}$$

1b) $W = \frac{\epsilon_0}{2}\left[\int E^2 d\tau + \oint V\vec{E}\cdot d\vec{a}\right]$

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$$

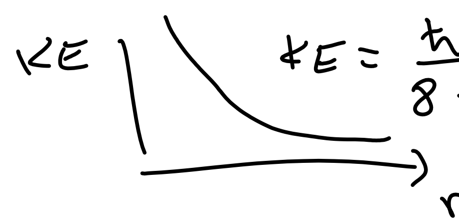
$$da = D^2\sin\theta\, d\theta\, d\phi$$

$$\int_0^R E_{in}^2\,d\tau + \int_R^D E_{out}\,d\tau \qquad E_{out} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

$$d\tau = 4\pi r^2\,dr$$

6.)

$PE = \frac{1}{4\pi\epsilon_0}\frac{-q^2}{r}$



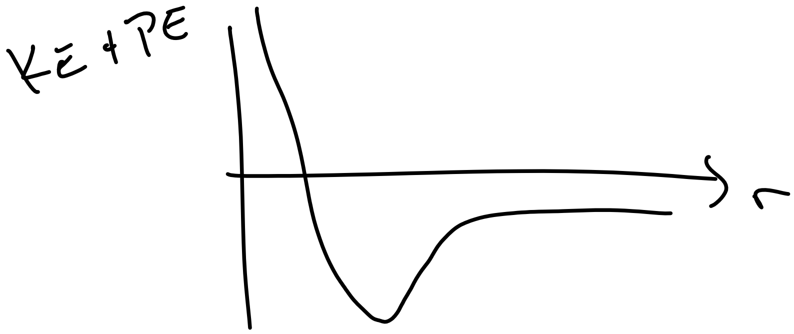
$KE = \frac{\hbar^2}{8r^2m}$

← well of width $r \to \Delta x$

$KE = \frac{p^2}{2m} \qquad \Delta p\Delta x \geq \frac{\hbar}{2}$

$p \to \Delta p \qquad \Delta p = \frac{\hbar}{2\Delta x} \to \frac{\hbar}{2r}$

$$KE + PE = \left(\frac{\hbar}{2r}\right)^2\Big/2m \quad \frac{1}{4\pi\epsilon_0}\left(-\frac{q^2}{r}\right)$$



Find value of r that gives minimum KE + PE

$$\frac{d}{dr}(KE + PE) = 0$$

solve for r