

1. (a) Using the integral form of Gauss's Law, find the electric field due to a infinite cylinder of charge. The cylinder has radius R and charge density $\rho = A/r$, where A is a constant. Make sure you find the field both inside and outside the cylinder. (b) Prove that your result is consistent with the differential form of Gauss's Law.

$$\oint \vec{E} \cdot d\vec{a} = \frac{\rho dV}{\epsilon_0}$$



$$\vec{E}_\perp d\vec{a} \cdot \vec{E} \cdot d\vec{a} = E 2\pi r L$$

$$Q_{\text{enc}} = \int \rho dV = \int_A^r \frac{A}{r} 2\pi r' L dr' r > R$$

$$r < R \quad E 2\pi r L = A \frac{2\pi L r}{\epsilon_0} \quad E = \frac{A}{\epsilon_0}$$

$$Q_{\text{enc}} = \int_0^R \frac{A}{r} 2\pi r' L dr' r > R$$

$$r > R \quad E 2\pi r L = A \frac{2\pi L R}{\epsilon_0} \quad E = \frac{A R}{\epsilon_0 r}$$

$$\vec{E}_0 \cdot \vec{E} = \rho/\epsilon_0 \quad \text{cylindrical coords} \quad \frac{1}{s} \frac{\partial}{\partial s} (s E_s) + \frac{\partial}{\partial \phi} + \frac{\partial}{\partial z} = \rho/\epsilon_0$$

$$r < R \quad \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{A}{\epsilon_0} \right) = \frac{A}{\epsilon_0} r = \rho/\epsilon_0 \quad \rho = A/r \quad \checkmark$$

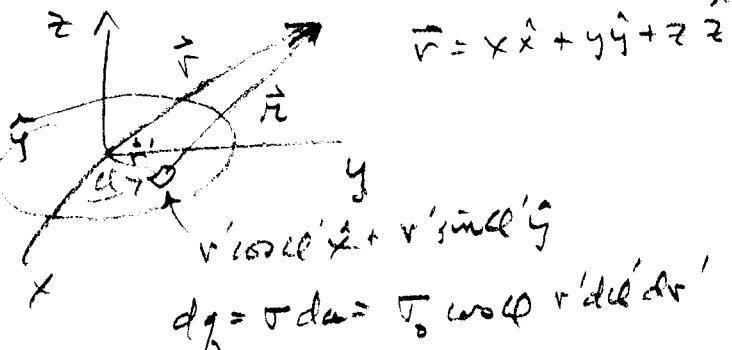
$$r > R \quad \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{A R^2}{\epsilon_0 r} \right) = 0 \quad \rho = 0 \quad \checkmark$$

2. Write an integral expression for the electric field at some arbitrary point due to a disk of charge of radius R located in the xy plane with the center of the disk at the origin. The charge distribution is given by $\sigma = \sigma_0 \cos(\phi)$.

Use cartesian since unit vectors come out of integral

$$\hat{r} = \hat{v} - \hat{r}' = (x - r' \cos\phi) \hat{x} + (y - r' \sin\phi) \hat{y} + z \hat{z}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \iint_D \frac{\sigma_0 \cos\phi' r' dr' d\phi'}{r'^3} \hat{r}$$



$$d\phi = \tau d\omega = \tau_0 \cos\phi' r' dr' d\phi'$$

3. Derive an expression for the voltage at $r < R$ inside a uniformly charged sphere of radius R using $V = 0$ at infinity. Please put your work on the back of this page.

Gauss's Law
 $\nabla \times \vec{E} = -\frac{1}{\epsilon_0} \vec{J}$
 $\Delta V = - \int \vec{E} \cdot d\vec{r}$

$$E_{\text{in}} 4\pi r^2 = \rho \frac{4\pi}{3} \frac{r^3}{\epsilon_0} \quad r < R$$

$$E_{\text{in}} = \frac{\rho r}{3\epsilon_0}$$

$$E_{\text{out}} 4\pi r^2 = \rho \frac{4\pi}{3} \frac{R^3}{\epsilon_0} \quad r > R$$

$$E_{\text{out}} = \frac{\rho}{3\epsilon_0} \frac{R^3}{r^2}$$

$$V_f - V_\infty = - \int_{\infty}^r E_{\text{out}} dr - \int_r^R E_{\text{in}} dr$$