Note Title 4/25/2007

-Equation sheet to be turned in Thursday 5 PM: Two sheets of paper but you can write on both sides. Only formulas allowed. No examples, problems, etc.
-Ask any question you have about the exam on the forum or see me.

On exam 4 I expect you to be able to
(1) use Ampere's law (in integral or differential form) with displacement current to find B given a change E.
(2) use Faraday's law (in integral or differential form) to find E given a change B.
(3) have a overview of the method of separation of variables (for example understand the logic in all the steps of solving a problem such as illustrated in section 3.3.1).
(4) calculate magnetic fields and currents in linear material using the H vector.
(5) Apply the divergence stokes theorems in both integral and differential form in the context of Maxwell's equations.

Maxwells eqns (free space) + Bndry cond.

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 \qquad \vec{\nabla}\cdot B = 0$$

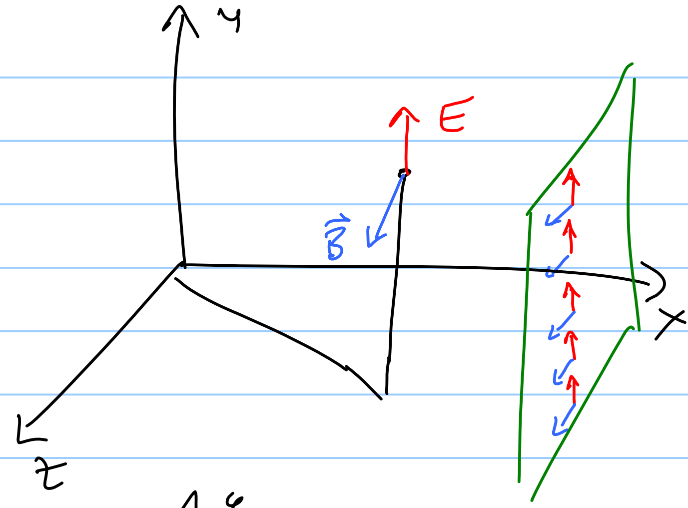
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \vec{\nabla}\times\vec{B} = \mu_0\Big(\underbrace{\vec{J}}_{\text{charges}} + \overbrace{\vec{J}_D}^{\epsilon_0 \frac{\partial \vec{E}}{\partial t}}\Big)$$

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t} \quad \text{cons. charge}$$

currents & charges generate fields & fields exert forces

$$\vec{F} = q\vec{v}\times\vec{B} + q\vec{E}$$

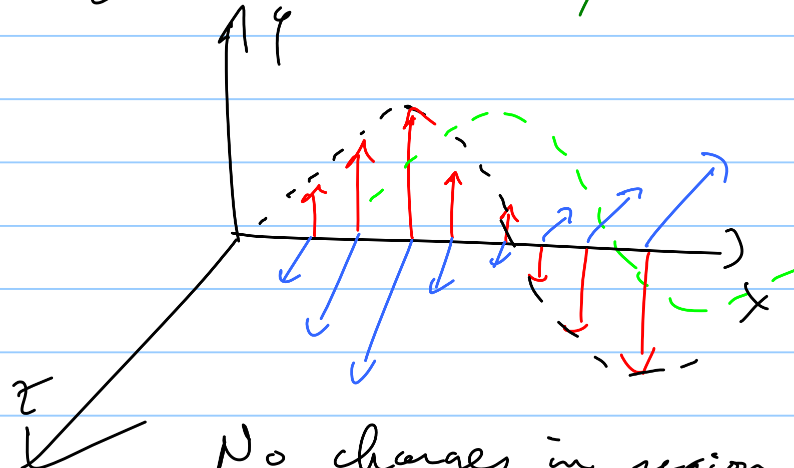
Ex: Plane EM wave



E & $\vec{B}$ are same for every pt in y-z plane

$$\vec{E} = E_0 \cos(kx - \omega t)\,\hat{y}$$

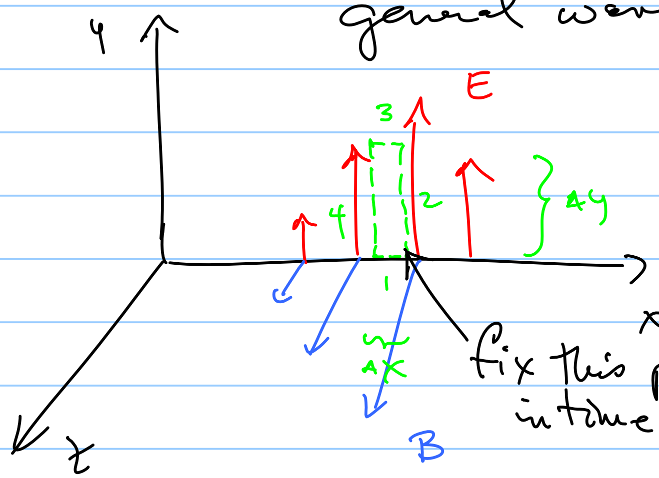
fix t



$$\vec{B} = B_0 \cos(kx - \omega t)\,\hat{z}$$

No charges in region

general waveform



fix this path in time while wave moves past this path

partial derivatives

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\underbrace{\int \vec{\nabla} \times \vec{E} \cdot d\vec{a}}_{\oint \vec{E} \cdot d\vec{l}} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

$$\oint \vec{E} \cdot d\vec{l} = \underbrace{\cancel{\int_1 \vec{E} \cdot d\vec{l}}^{\,0}}_{E_\perp dl} + E_y(x_2)\Delta y + \cancel{\int_3}^{\,0} - E_y(x_1)\Delta y$$

$$E_y(x_2) \simeq E_y(x_1) + \frac{\partial E_y}{\partial x}\Delta x$$

$$\oint \vec{E}\cdot d\vec{l} = \left(E_y(x_1) + \frac{\partial E_y}{\partial x}\Delta x\right)\Delta y - E_y(x_1)\Delta y = \frac{\partial E_y}{\partial x}\Delta x\Delta y$$

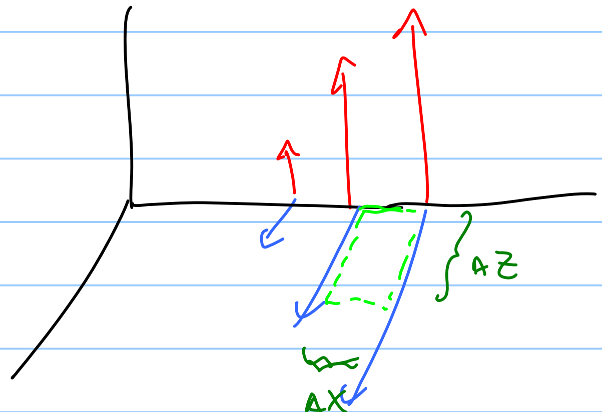
$$-\frac{\partial}{\partial t}\underbrace{\int \vec{B}\cdot d\vec{a}}_{-\frac{\partial}{\partial t}B_z\Delta x\Delta y} = -\frac{\partial B_z}{\partial t}\Delta x\Delta y = \frac{\partial E_y}{\partial x}\Delta x\Delta y$$

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

1st order PDE

differentiate w.r.t t

Apply ampere's law to this situation



$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial \vec{E}}{\partial t}$$

$$\underbrace{\int \vec{\nabla}\times\vec{B}\cdot d\vec{a}}_{\oint \vec{B}\cdot d\vec{l}} = \mu_0\epsilon_0\frac{\partial}{\partial t}\int \vec{E}\cdot d\vec{a}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\epsilon_0\frac{\partial}{\partial t}E_y\Delta x\Delta z$$

$$-B_z(x_2)\Delta z + B_z(x_1)\Delta z$$

$$B_z(x_2) = B_z(x_1) + \frac{\partial B_z}{\partial x}\Delta x$$

$$-\frac{\partial B_z}{\partial x}\Delta x\Delta z = \mu_0\epsilon_0\frac{\partial E_y}{\partial t}\Delta x\Delta z$$

$$\boxed{-\frac{\partial B_z}{\partial x} = \frac{\partial E_y}{\partial t}}$$

differentiate w.r.t x

$$\frac{\partial^2 E_y}{\partial t\,\partial x} = \mu_0\epsilon_0\frac{\partial^2 B_z}{\partial t^2} \qquad -\frac{\partial^2 B_z}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 E_y}{\partial x\,\partial t}$$

equal $\qquad$ equal

$$\boxed{\frac{\partial^2 B_z}{\partial x^2} = -\mu_0\epsilon_0\frac{\partial^2 B_z}{\partial t^2}}$$

wave eqns for E_y & B_z

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

Similar procedure yields $\frac{\partial^2 E_y}{\partial x^2} = -\mu_0\epsilon_0\frac{\partial^2 E_y}{\partial t^2}$