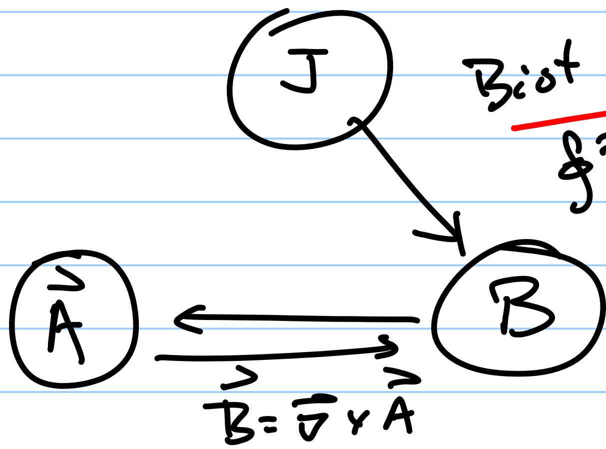
Overview

Note Title 3/21/2008

$$\vec{F} = q\vec{v}\times\vec{B} \xrightarrow[\text{wire}]{} \int \underbrace{I\,d\vec{l}'\times\vec{B}}_{\vec{I}\times\vec{B}\,dl'} \xrightarrow[\text{sheet}]{\text{current}} \int \vec{K}\times\vec{B}\,da' \longrightarrow \int \vec{J}\times\vec{B}\,d\tau'$$

Find $\vec{B}$ magnetostatics: $\underline{\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,d\hat{l}\times\hat{r}}{r^2}}$

$$\longrightarrow \frac{\mu_0}{4\pi}\int \frac{\vec{K}\times\hat{r}}{r^2}\,da \longrightarrow \frac{\mu_0}{4\pi}\int \frac{\vec{J}\times\hat{r}}{r^2}\,d\tau'$$



Biot Savart

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$

V

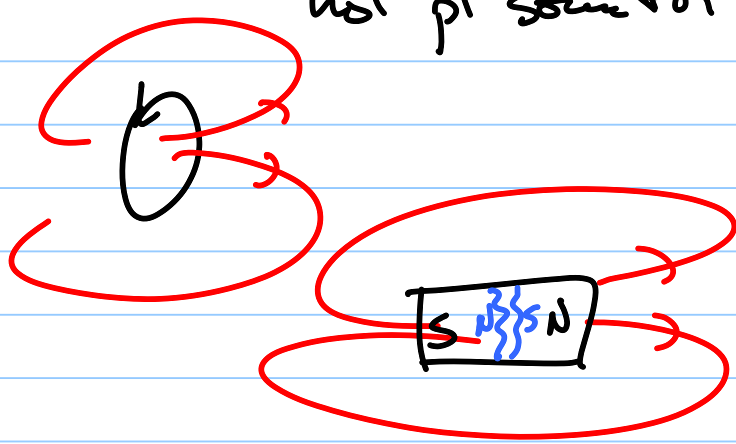
ρ $E = \int \frac{k\,dq}{r^2}\hat{r}$

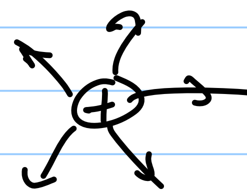
$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$

$\vec{E}$

Biot Savart $\Rightarrow$ $\vec{\nabla}\cdot\vec{B} = 0$ $\qquad \vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$

not pt source for B

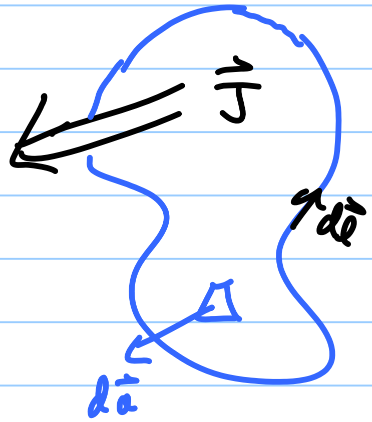




$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$ diff form of Biot Savart

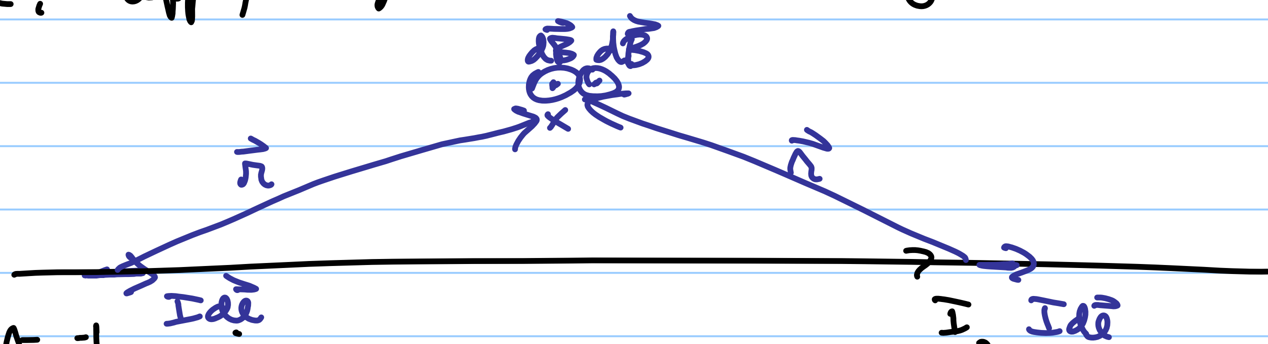
$$\int \overbrace{\vec{\nabla}\times\vec{B}}^{\mu_0 \vec{J}} \cdot d\vec{a} = \oint \vec{B}\cdot d\vec{l}$$

tiles that make up surface



$$\mu_0 \underbrace{\int \vec{J}\cdot d\vec{a}}_{I_{enc}} = \oint \vec{B}\cdot d\vec{l}$$ Amperes Law

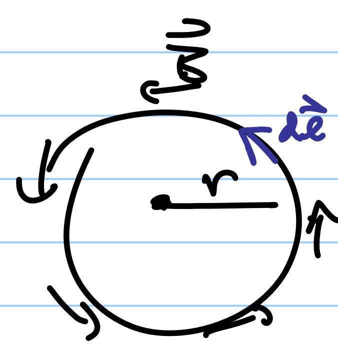
Ex: apply Amps Law to find B give I_0



infinite wire

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

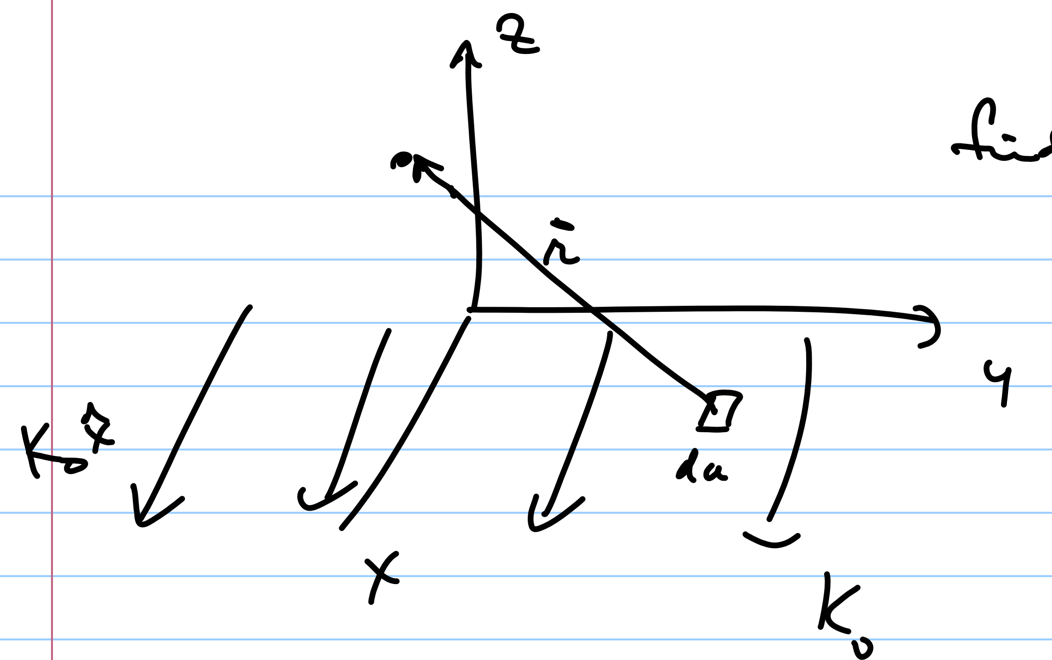
$$\oint |\vec{B}||d\vec{l}| \underbrace{\cos\phi}_{1}$$



endview

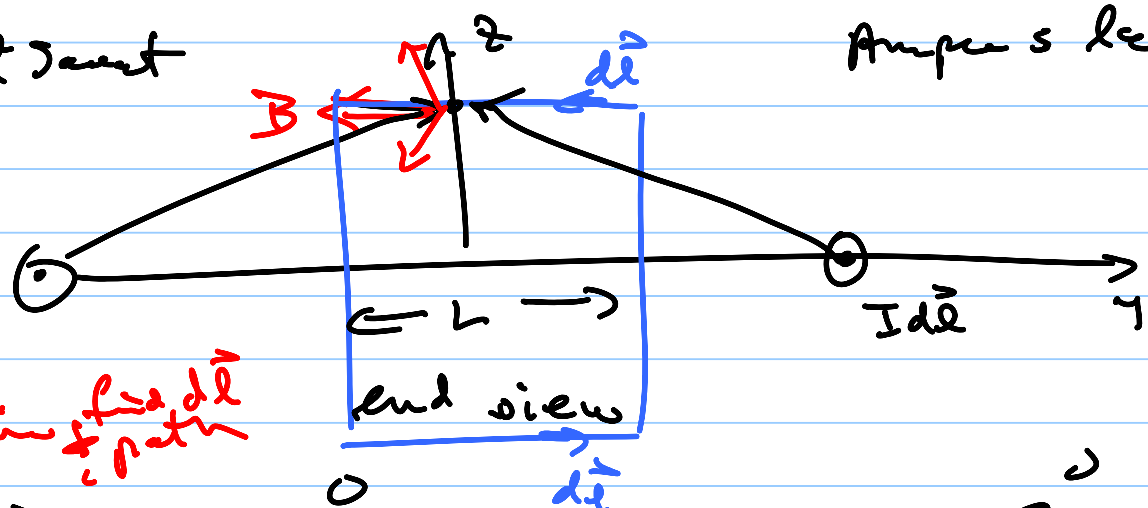
$$B \oint |d\vec{l}| = B\, 2\pi r = \mu_0 I_0 \qquad \boxed{B = \frac{\mu_0 I_0}{2\pi r}}$$

find B above & below xy plane



(1) Biot Savart

Ampere's law



(2) outline & path find $d\vec{l}$

$$\oint \vec{B}\cdot d\vec{l} = \int_{sides}^{\to 0} + \int_{top} + \int_{bottom} + \int_{side}^{\to 0}$$

$$B\cancel{L} + B\cancel{L} = \mu_0 K \cancel{L}$$

$$B = \frac{\mu_0 K}{2}$$

(3) limits $K \to 0$: $B \to 0$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0 \qquad \vec{E} = -\vec{\nabla} V \Rightarrow \nabla^2 V = -\rho/\epsilon_0$$

$$\cancel{\vec{B} = \vec{\nabla} \varphi}$$

$$\vec{\nabla} \times \vec{B} = \vec{\nabla} \times \vec{\nabla} \varphi = \emptyset \text{ always}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{\nabla} \times \vec{A} = \emptyset \qquad \text{always zero}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \times \vec{A} = \vec{\nabla} \times \vec{A}' + \underbrace{\vec{\nabla} \times \vec{\nabla} \psi}_{\text{always zero}}$$

$\vec{A}$ is not unique up to a const. but also we can add $\vec{\nabla}\psi$ and nothing changes

Want to find $\vec{A}$ we need $\boxed{\vec{\nabla}\times\vec{A} = \vec{B}}$ & $\vec{\nabla}\cdot\vec{A}$

$$\vec{\nabla}\cdot\vec{A} = \vec{\nabla}\cdot\vec{A}' + \underbrace{\nabla^2 \varphi}_{\text{can be anything}}$$

choice of $\nabla^2\varphi \Rightarrow \boxed{\vec{\nabla}\cdot\vec{A} = 0}$

Gauge choice: Coulomb gauge

Ex: $B = B_0\hat{k}$ given find $\vec{A}$

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = B_0\hat{k}$$

$$\vec{A} = -\frac{B_0 y}{2}\hat{x} + \frac{B_0 x}{2}\hat{y} + 0\hat{z}$$

$$\vec{A} = -B_0 y$$