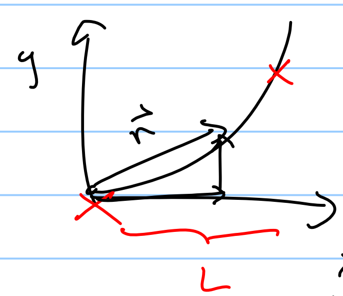
Note Title 2/8/2008

Review Exam I

- divergence th. : do flux integral
- stokes th : line integral



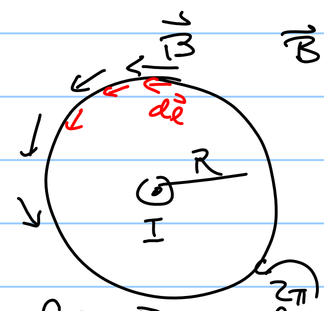
$y = \alpha x^2$

$\vec{r} = x\hat{x} + y\hat{y}$

$\vec{r} = x\hat{x} + \alpha x^2 \hat{y}$

$d\vec{l} = d\vec{r} = dx\,\hat{x} + 2\alpha x\, dx\,\hat{y}$

Arc length $= \int_0^L |d\vec{r}|$

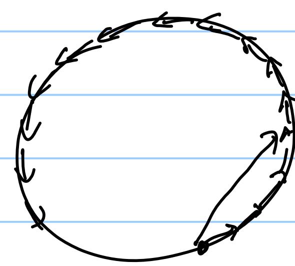


$\vec{B}\cdot d\vec{l} = |\vec{B}||d\vec{l}| \times \cos\phi$

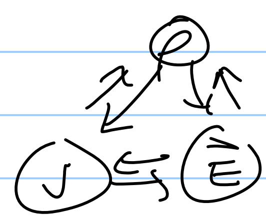
$= |\vec{B}|\, R\, d\theta$

$\oint \vec{B}\cdot d\vec{l} = B\int_0^{2\pi} R\, d\theta = B\, 2\pi R$

$\oint d\vec{l} = \emptyset$

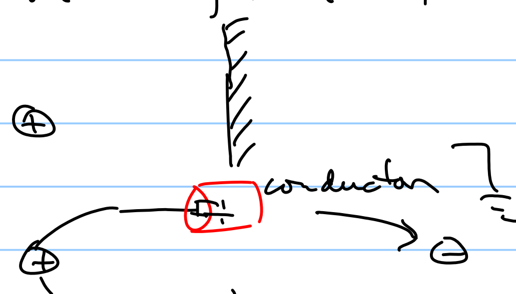


- triangle diagram



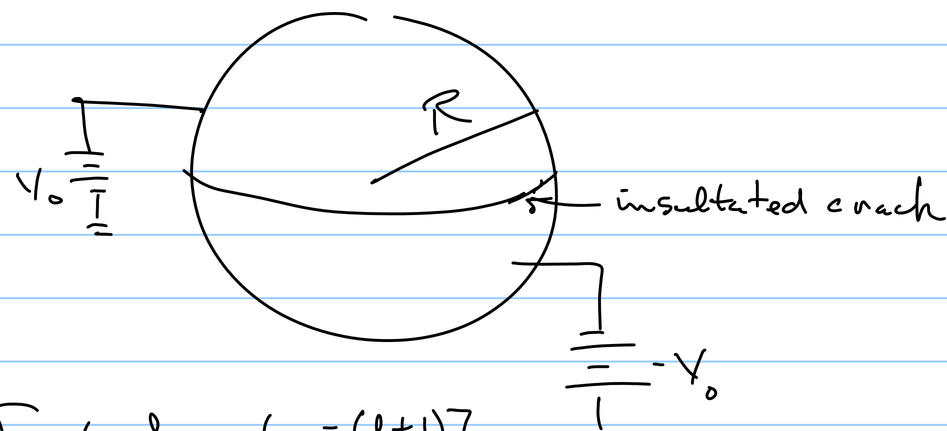
$\boxed{\text{Summation problem}}$

- energy in fields/charge distribution
- conductors : bndry condition $E_\perp$ & V const.
- Laplace's eqn bndry value problem
  (give V on bndry find V & σ everywh)
  - relaxation method
  - images
  - sep. variables



- cartesian coords

- spherical coords (no φ dep)



General soln

$$V(r,\theta) = \sum_{\ell=0}^{\infty}\left[A'_\ell r^\ell + B'_\ell r^{-(\ell+1)}\right]P_\ell(\cos\theta)$$

$$V_{in} = \sum A'_\ell P_\ell r^\ell \qquad V_{out} = \sum B'_\ell r^{-(\ell+1)} P_\ell$$

at $r = R$ $\quad V_{in}\big|_R = V_{out}\big|_R$

$$A'_\ell R^\ell \cancel{P_\ell} = B'_\ell R^{-(\ell+1)} \cancel{P_\ell}$$

$$\underline{A'_\ell R^\ell} = B'_\ell R^{-(\ell+1)} \equiv A_\ell$$

define

$$\begin{cases} A'_\ell = \dfrac{A_\ell}{R^\ell} \\ B'_\ell = \dfrac{A_\ell}{R^{-(\ell+1)}} = A_\ell R^{(\ell+1)} \end{cases}$$

$$V_{in} = \sum A_\ell \left(\frac{r}{R}\right)^\ell P_\ell \qquad V_{out} = \sum A_\ell \left(\frac{R}{r}\right)^{\ell+1} P_\ell$$

Set V_{in} (or V_{out}) $= \sum A_\ell P_\ell$ multiply both sides by P_m & integrate

$$\int_{-1}^{1} V_0 \underbrace{f(\theta)}_{\text{square wave}} P_m \, d(\cos\theta) = \sum_l A_l \int_{-1}^{1} P_l P_m \, d(\cos\theta)$$

$$= \sum_l A_l \frac{2\delta_{lm}}{2l+1}$$

m even is even function

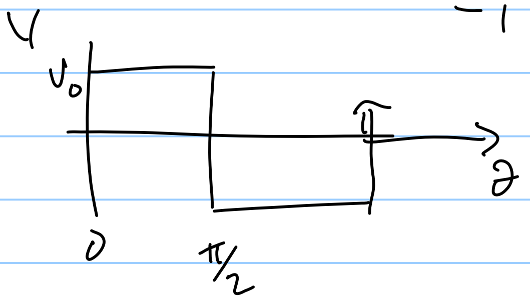
$$\Rightarrow A_l = \frac{2l+1}{2} V_0 \int_{-1}^{1} \underbrace{f}_{\text{square wave}} P_m\underbrace{(\cos\theta)}_{x} \, d(\cos\theta)$$

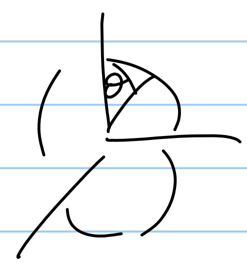
odd

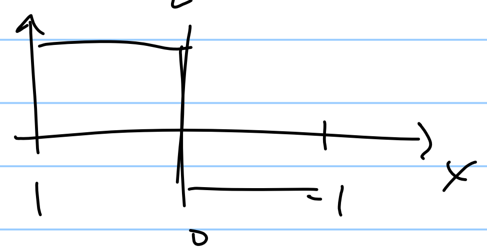
$x = \cos\theta$

$x = 1 \quad \theta = 0$

$x = -1 \quad \theta = \pi$







for P_m odd $\quad A_l = 2 \frac{(2l+1)}{2} V_0 \int_0^1 P_m(\cos\theta) \, d(\cos\theta)$

$$A_1 = 3V_0 \int_0^1 \cos\theta \, d(\cos\theta) = \frac{3}{2} V_0$$

$x = \cos\theta \qquad dx = d(\cos\theta)$

$A_3 =$

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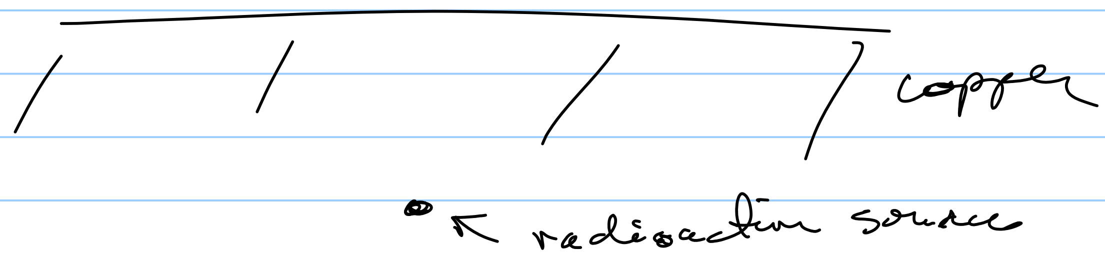
thermal statics

$$\nabla^2 T \propto \frac{\partial T}{\partial t} = 0$$

No sources or sinks of heat energy

$T(x,y,z)$ obeys Laplaces eqn

For a source of thermal energy in thermal equilibrium



$$\nabla^2 T \propto \rho_{\text{nuclear}}$$

$$\nabla^2 V = \rho/\epsilon_0 \leftarrow \text{source of } \vec{E} \text{ \& } V$$