Some solutions for 4/16/08

Note Title 4/16/2008

$$\nabla^2 f(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)f(r) = 0$$

$$\Rightarrow \quad \frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) = 0$$

$$\Rightarrow \quad r^2 \frac{\partial f}{\partial r} = \text{Const}$$

$$\Rightarrow \quad \frac{\partial f}{\partial r} = \frac{c_0}{r^2}$$

$$\Rightarrow \quad \boxed{f(r) = -\frac{c_0}{r} + c_1}$$

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$l=0 \qquad R(r) = B\,N_0(kr)$

$N_0(x) = -\dfrac{\cos x}{x}$ so

$R(a) = -\dfrac{\cos(ka)}{ka} = 0$

$\Rightarrow ka = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \ldots$

$\left(\dfrac{2N-1}{2}\right)\pi$

$\Rightarrow k_N = \dfrac{2N-1}{2}\dfrac{\pi}{a}$

Since $k = \sqrt{2mE}/\hbar$

$\Rightarrow \dfrac{k_N^2\hbar^2}{2m} = E_N = \dfrac{\hbar^2}{2m}\left(\dfrac{2N-1}{2}\right)^2\dfrac{\pi^2}{a^2}$

$$\underset{\substack{\downarrow \\ l=0}}{E_{N0}} = \frac{\hbar^2\pi^2}{8ma^2}(2N-1)^2$$

$E_{10} = \dfrac{\hbar^2\pi^2}{8ma^2} \qquad E_{20} = \dfrac{9}{8}\dfrac{\hbar^2\pi^2}{ma^2} \qquad \ldots$

for $r \le a$ $u(r) = A r j_0(kr)$

$= A \sin(kr)$

with $k = \frac{\sqrt{2m(E+V_0)}}{\hbar}$

for $r > a$ $V=0$, and for $l=0$
the TISE is

$$\frac{d^2u}{dr^2} = -\frac{2m}{\hbar^2} E u \equiv K^2 u$$

$$\Rightarrow \quad \frac{d^2u}{dr^2} - K^2 u = 0$$

why the minus sign

so $u(r) = \underbrace{\cancel{Ce^{Kr}}}_{0} + De^{-Kr}$

$$\Rightarrow \quad u(r) = De^{-Kr}$$

B.C. are continuity of u + u'
at $r=a$

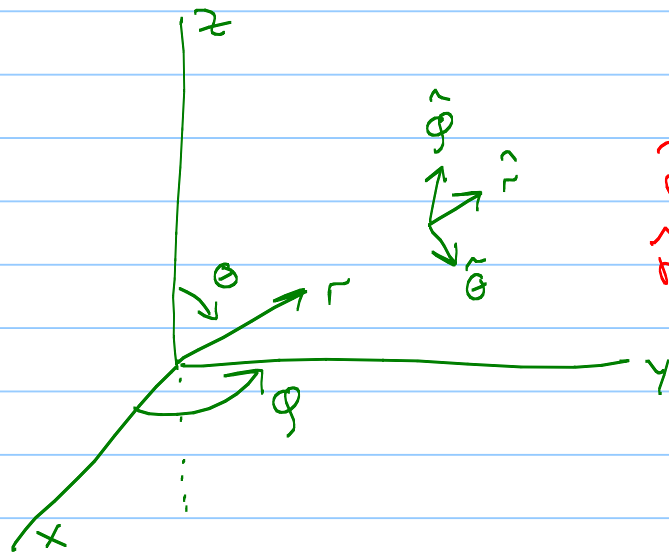
$$\boxed{A\sin(ka) = De^{-Ka}}$$

$$\boxed{Ak\cos(ka) = -Dke^{-ka}}$$

$$\hat{\vec{L}} = -i\hbar\, \vec{r} \times \nabla$$

$$= -i\hbar\, \vec{r} \times \left[\hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\right]$$

$$= -i\hbar\left[\underbrace{r\,\hat{r}\times\hat{r}}_{0}\frac{\partial}{\partial r} + \underbrace{\hat{r}\times\hat{\theta}}_{\hat{\phi}}\frac{\partial}{\partial \theta} + \underbrace{\hat{r}\times\hat{\phi}}_{-\hat{\theta}}\frac{1}{\sin\theta}\frac{\partial}{\partial \phi}\right]$$



$\hat{r} \times \hat{\theta} = \hat{\varphi}$

$\hat{r} \times \hat{\varphi} = -\hat{\theta}$

$$\vec{L} = -i\hbar\left[\hat{\varphi}\frac{\partial}{\partial \theta} - \hat{\theta}\frac{1}{\sin\theta}\frac{\partial}{\partial \varphi}\right]$$

$$L^2 = \vec{L}\cdot\vec{L} =$$

$$\vec{L} f(\theta,\varphi) = -i\hbar\left[\hat{\varphi}\frac{\partial f}{\partial \theta} - \hat{\theta}\frac{1}{\sin\theta}\frac{\partial f}{\partial \varphi}\right]$$

Didn't as this but

$$L^2 = -\hbar^2 \left[\hat{\varphi} \frac{\partial}{\partial\theta} - \hat{\theta} \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \right] \left[\hat{\varphi} \frac{\partial f}{\partial\theta} - \hat{\theta} \frac{1}{\sin\theta} \frac{\partial f}{\partial\varphi} \right]$$

but be careful, the unit vectors in spherical coord. are <u>Not</u> constant

$\frac{\partial\hat{\theta}}{\partial\varphi}$, $\frac{\partial\hat{\theta}}{\partial\theta}$, ... all need to be computed.

$$Q = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$\hat{e}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\qquad$ $\hat{e}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\lambda_1 = 3$ $\qquad$ $\lambda_2 = -1$

$$\hat{e}_1 \hat{e}_1^T = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\hat{e}_2 \hat{e}_2^T = \frac{1}{2} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$3 \, \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$