PHGN 462 Homework 2

1a) Write down the complete Maxwell equations in differential form. Then describe, in words and sentences, what each of them is saying. And by that I mean talk about the physical content – your answer should not be of the form "This one is saying that del dot E equals rho over epsilon naught." If an equation includes multiple kinds of source (like free and bound), indicate them separately.

b) Now write down the Maxwell equations in integral form. Clearly indicate which integrals are over lines, areas, or volumes, and clearly indicate which domains are open and which are closed. Also note that there's more than one acceptable integral form of Faraday's law; use what you like.

c) Show me how to get from the differential form of Faraday's law to the integral form. If you're restricting the situation somehow, make sure you explain how and why. Then show me how to get from the integral form to the differential form. Explain all the steps.

d) Show me how the Maxwell equations need to change if we want to accommodate magnetic monopoles. Explain the changes. We did this in class, and you should feel free to refer to that, but take the time to understand what you're doing and to describe it in your own words.

From this point on, I'm going to assume that you're fluent in the Maxwell equations, so make sure you've got this all down (not so much the magnetic monopole stuff, but the other stuff definitely).

2) Let's suppose we have a capacitor with circular parallel plates of radius *a* and separation *d*. There's some time-dependent potential difference *V(t)* across the plates.

a) Show that the magnetic field in between the plates some distance *r* from the centerline (for *r* > *a*) is

$$B(r) = \mu_0 \varepsilon_0 \frac{a^2}{2rd} \frac{dV}{dt}$$

b) Show that B is the same as the field of a straight wire carrying current $I = \frac{dQ}{dt}$, where Q is the charge on the capacitor

3a) Let's start with a straight infinite wire of with some steady current I running through it. Write down the magnetic field it makes. Note that this is an idealized 1-D, zero-radius wire.

b) Now calculate the energy contained in that magnetic field. Or, more precisely, calculate the energy per meter of wire, since it's infinitely long.

c) If you did it right, something should have gone terribly, terribly wrong in part b, owing to the fact that a 1-D wire is not an entirely realistic model of a wire. This happens sometimes. Models work until you push on them too hard, and then they break. Now model the wire as having some finite radius a, as wires generally do, and calculate the new fields and energy per meter. Does that fix it? Comment one way or the other.

d) Now find the energy contained in the electric field of an electron.

e) You probably got something divergent again, if you tried to treat an electron as a point object. The thing is, an actual observed electron (as opposed to the wavefunction describing where it might be) *is* a radius-zero point, as far as we can tell. But there *are* error bars on that zero. Read up on what we know about the size of an electron, and see if you can fix things so that you can get a finite result for the electron's field energy. Go ahead and plug in numbers, too, to make sure this doesn't generate something absurd like 10^{100} Joules or something. Also, be aware that the so called 'classical electron radius' does <u>not</u> represent a measurement of the electron's radius.

Note: Self-energies are notoriously tricky things. Infinities show up a lot and, frankly, it can be hard to get rid of them. See for example the whole notion of renormalization in quantum field theory.

4) I think by now everyone has seen a Tesla coil. And, as it turns out, we now know enough to completely understand their operation. The most common design is basically a couple transformers and a capacitor (later designs involve vacuum tubes and transistors and such, but let's not worry about those).

Sketch out a design for a Tesla coil that'll make foot-long sparks when plugged into a standard 120V 60Hz wall outlet. Tell me what kind of inductors, capacitors, transformers, etc need to go in there, in which arrangement. I'd like a schematic and also a description of the parts and why they're there.

I'm deliberately leaving this rather open. It's a design task. That's kind of how those are. But correspondingly, I invite you to look up whatever you want in books or on the internet or wherever. Just get it done and tell me how.