MATH 348 - Advanced Engineering Mathematics April 21, 2008
Homework 9 - Hints

Definition - Linear Combination - Given a set, S, of n vectors,

$$S = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \ldots, \mathbf{a}_n\}. \tag{1}$$

We say that $\mathbf{b}$ is a linear combination of the vectors from S if there exist scalars $x_1, x_2, x_3, \ldots, x_n$ such that,

$$\mathbf{b} = \sum_{i=1}^{n} x_i \mathbf{a}_i. \tag{2}$$

Definition - Linear Independence - Given a set, S, of n vectors,

$$S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_n\}. \tag{3}$$

We say that the vectors of S forms a linearly independent set if and only if the scalars $x_1 = x_2 = x_3 = \cdots = x_n = 0$ are the only solution to,

$$\mathbf{0} = \sum_{i=1}^{n} x_i \mathbf{v}_i. \tag{4}$$

If a set of vectors does not form a linearly independent set then the vectors are said to be linearly dependent.

Definition - Column Space - Given the matrix $\mathbf{A} \in \mathbb{R}^{m\times n}$ we define the set of all linear combinations of the columns of the matrix $\mathbf{A}$ to be the column space of $\mathbf{A}$ and we denote this space of vectors as Col($\mathbf{A}$).

Definition - Null Space - Given the matrix $\mathbf{A} \in \mathbb{R}^{m\times n}$ we define the set of vectors $\mathbf{x}$, which satisfies the equation $\mathbf{Ax} = \mathbf{0}$ to be the Null Space of $\mathbf{A}$ and we denote this space of vectors Nul($\mathbf{A}$).

Note - Straightforward calculations will verify that,

$$\sum_{i=1}^{n} x_i \mathbf{a}_i = x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \\ \vdots \\ a_{m2} \end{bmatrix} + x_3 \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \\ \vdots \\ a_{m3} \end{bmatrix} + \cdots + x_n \begin{bmatrix} a_{1n} \\ a_{2n} \\ a_{3n} \\ \vdots \\ a_{mn} \end{bmatrix} = \mathbf{Ax} \tag{5}$$

Hints:

- Problem 3 - If you are given $\mathbf{Ax} = \mathbf{b}$ and a solution $\mathbf{x}$ exists then what does this imply about $\mathbf{b}$ in light of (2)?
- Problem 4 - If $\mathbf{x} = \mathbf{0}$ is the only solution to $\mathbf{Vx} = \mathbf{0}$ then (5) implies that the columns of $\mathbf{V}$ forms a linearly independent set.
- Problem 5 - What is $\mathbf{Aw}$? What does this imply about $\mathbf{w}$? Is there a solution to $\mathbf{Ax}$=$\mathbf{w}$? What does this imply about $\mathbf{w}$?