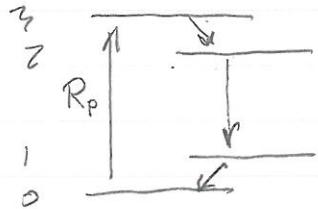


## Saturation

gain saturation in a 4-level system



assume  $\tau_{32} \ll \tau_{21}$ ,  $W_2, N_2$   
 $\tau_{10} \ll \dots$

look at level 2 only

$$\frac{dN_2}{dt} = +R_p - W N_2 - \frac{\perp}{\tau_{21}} N_2$$

pump      stim      spont.

in steady state,  $dN_2/dt = 0$

$$\Rightarrow N_2 = \frac{R_p}{W + \frac{\perp}{\tau_{21}}} = \frac{R_p \tau}{1 + W \tau_{21}}$$

remember  $W \propto I$

$$W = \frac{c I}{h\nu}$$

$$W \tau_{21} = I \cdot \frac{c \tau_{21}}{h\nu} = \frac{I}{I_s}$$

saturation intensity:

$$I_s = \frac{h\nu}{c \tau_{21}} \quad \text{1 photon / atom / time}$$

$$N_2 = \frac{N_{20}}{1 + I/I_s} \quad N_{20} = R_p \tau$$

gain saturation:

$$g = \frac{g_0}{1 + I/I_s}$$

saturation with 2 levels

- no external pumping

$$\frac{dN_2}{dt} = -WN_2 + W\frac{g_1}{g_2}N_1 - \frac{N_2}{\tau}$$

$$\frac{dN_1}{dt} = -\frac{dN_2}{dt} \quad \text{closed 2-level system}$$

change variables: for simplicity let  $g_1 = g_2$   
total # density

$$N_t = N_1 + N_2 \quad (\text{constant})$$

difference

$$\Delta N = N_1 - N_2$$

combine -

$$N_t = \Delta N + 2N_2 \rightarrow N_2 = \frac{1}{2}(N_t - \Delta N)$$

$$\frac{d}{dt}(N_1 - N_2) = -2 \frac{dN_2}{dt}$$

$$\frac{d}{dt}\Delta N = 2W(N_2 - N_1) + \frac{2N_2}{\tau}$$

$$= -2W\Delta N + \frac{N_t}{\tau} - \frac{1}{\tau}\Delta N$$

$$= -\Delta N(2W + \frac{1}{\tau}) + \frac{1}{\tau}N_t$$

At steady state,  $\frac{d}{dt} \rightarrow 0$

$$\Rightarrow \Delta N = \frac{N_t}{1 + 2W\tau}$$

## Saturation effects

2 levels, absorbing transition, constant intensity

$$\frac{dN_2}{dt} = -W(N_2 - N_1) - \frac{N_2}{\tau}, \quad \frac{dN_1}{dt} = -\frac{dN_2}{dt} \quad \text{closed 2-level system}$$

$$\begin{aligned} \text{write in terms of: total } N_t &= N_1 + N_2 \\ \text{diff } \Delta N &= N_1 - N_2 \end{aligned} \quad \left. \begin{aligned} N_t &= \Delta N + 2N_2 \\ \Delta N &= N_1 - N_2 \end{aligned} \right\} \rightarrow N_2 = \frac{1}{2}(N_t - \Delta N)$$

$$\frac{d}{dt}(N_1 - N_2) = -2 \frac{dN_t}{dt}$$



$$\begin{aligned} \frac{d}{dt} \Delta N &= -2W\Delta N + 2\frac{N_2}{\tau} = -2W\Delta N + \frac{N_t}{\tau} - \frac{\Delta N}{\tau} \\ &= -\Delta N\left(\frac{1}{\tau} + 2W\right) + \frac{1}{\tau}N_t \end{aligned}$$

in steady state:  $\frac{d}{dt} \rightarrow 0$

$$\Delta N = \frac{N_t}{1 + 2W\tau}$$

$W\tau$  is the key parameter:  $W\tau \gg 1 \quad \Delta N \rightarrow 0 \quad \text{or} \quad N_2 = N_1$

Net absorbed power/volume  $\frac{dP}{dV}$

$$\frac{dP}{dV} = h\nu W\Delta N = h\nu \frac{N_t W}{1 + 2W\tau}$$

$$\text{as } W\tau \gg 1 \quad \frac{dP}{dV} \rightarrow h\nu \frac{N_t}{2} \cdot \frac{1}{\tau}$$

Radiated power:  $\frac{N_t}{2}$  lose  $h\nu$  in time  $\tau$

Saturation intensity:

$\sigma$  = abs. cross-section

$\sigma I$  = absorbed power per atom

$\frac{\sigma I}{h\nu}$  =  $W$  absorption rate.

In steady state:

$$\frac{\Delta N}{N_t} = \frac{1}{1 + 2\omega_0} = \frac{1}{1 + \frac{2\sigma c}{h\nu} I}$$

$\underbrace{\quad}_{1/I_s}$

Saturation intensity:

$$I_s = \frac{h\nu}{2\sigma} \rightarrow \text{steady state } \frac{\Delta N}{N_t} = \frac{1}{1 + I/I_s}$$

In terms of absorption:

$$\alpha = \frac{\omega_0}{1 + I/I_s} \quad \text{as } I \uparrow \rightarrow \text{less absorption.}$$

At  $I = I_s$  stim. emission rate = spontaneous em. rate.

Note:  $\omega_0$  depends on  $\nu$

$\alpha(\nu, I)$  doesn't change shape during saturation.

(constant  $\nu$  of pump beam)

If input  $\nu$  isn't at  $\nu_0 \rightarrow$  smaller  $\sigma$ , larger  $I_s$

Pulsed input  $I = I(t)$  duration  $\tau_p$

if  $\tau_p \gg \tau$  (lifetime) and pk  $I \ll I_s$ ,

$\frac{d\Delta N}{dt} \ll N_t/\tau$   $\rightarrow$  similar to CW behavior  
 $\Delta N(t)$  tracks  $I(t)$ .

for  $\tau_p \ll \tau$  then we can drop  $1/\tau$  terms

$$\frac{d}{dt} \Delta N \doteq -2\gamma \Delta N = -\frac{2\sigma}{h\nu} I(t) \Delta N$$

$$\text{integrate } \Delta N(t) = N_t \exp \left[ -\frac{2\sigma}{h\nu} \underbrace{\int_0^t I(\tau) d\tau}_{\text{energy fluence } \Gamma(t)} \right]$$

medium just integrates pumping pulse

define saturation fluence:

$$\Gamma_s = \frac{h\nu}{2\sigma}$$

absorption coeff

$$\alpha = \frac{2\pi^2}{3\mu_{\text{EoC}}} |\mu|^2 \Delta N \propto g(\nu - \nu_i) \propto \Delta N$$

$$\text{so since } \Delta N(t) = N_t e^{-\Gamma(t)/\Gamma_s}$$

$$\alpha(t) = \alpha_0 e^{-\Gamma(t)/\Gamma_s}$$

## Saturation fluence and pulsed input

For pulsed amplifiers, both pump and seed pulses are typically shorter in duration than the lifetime of the upper level.

rewrite eqn for  $\Delta N(t)$ : (2-level system)

$$\frac{d}{dt} \Delta N = -\Delta N \left( \frac{2\sigma I(t)}{\hbar\nu} + \frac{1}{\tau} \right) + \frac{1}{\tau} N_t$$

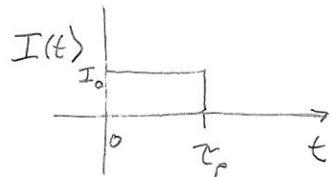
how to simplify equation?

$$\begin{aligned} \text{suppose } I(t) &= I_0 \text{ for } 0 < t < \tau_p \\ &= \Gamma_0 / \epsilon_p \end{aligned}$$

where  $\Gamma$  = energy fluence  $J/cm^2$

$$\rightarrow \frac{d}{dt} \Delta N = -\Delta N \left( \frac{2\sigma}{\hbar\nu} \Gamma_0 \frac{1}{\epsilon_p} + \frac{1}{\tau} \right) + \frac{1}{\tau} N_t$$

$\underbrace{\quad}_{\text{dimensionless}}$



- define saturation fluence  $\Gamma_s = h\nu / 2\sigma$
- first look at terms in  $( )$

if  $\frac{\Gamma_0}{\Gamma_s} \frac{1}{\epsilon_p} \gg \frac{1}{\tau}$  then we can ignore  $\frac{1}{\tau}$  term.

ex. Ti:Sapphire  $\Gamma_s \approx 1 J/cm^2$   
 $\epsilon_p \approx 3 \mu s$

for  $\tau_p = 10 ns$  (Flashlamp-pumped Nd:YAG)

2 terms are equal if:

$$\Gamma_0 = \Gamma_s \frac{\tau_p}{\epsilon_p} = \frac{1}{300} \Gamma_s \approx 3 mJ/cm^2$$

for  $\tau_p = 200 ns$  (arc-lamp-pumped Nd:YLF)

$$\Gamma_s = \Gamma_0 \cdot \frac{2}{30} \approx 67 mJ/cm^2$$

dropping that  $N_t$  term,

$$\frac{d}{dt} \Delta N = -\frac{\Gamma_0}{\Gamma_s} \frac{1}{\tau_p} \Delta N + \frac{N_t}{\tau}$$

now compare remaining two:

$$\text{at } t=0 \quad \Delta N = N_1 - N_2 = N_1 = N_t \quad (\text{all in level 1})$$

so we are still comparing  $\frac{\Gamma_0}{\Gamma_s} \frac{1}{\tau_p}$  to  $\frac{1}{\tau}$  as before.

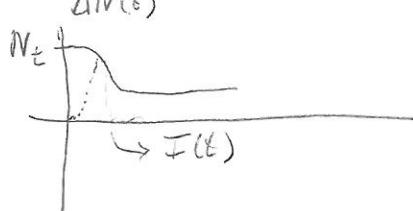
at later time  $\Delta N$  gets small

make ansatz (assumption) that we can drop  $N_t$ ,  
 then check our solution to see if our assumption is valid.

$$\frac{d}{dt} \Delta N = -\frac{I(t)}{\Gamma_s} \Delta N \quad \text{now integrate}$$

$$\ln \Delta N = -\frac{1}{\Gamma_s} \int_0^t I(t') dt'$$

$$\Delta N(t) = N_t \exp \left( -\frac{1}{\Gamma_s} \int_0^t I(t') dt' \right)$$



$\Delta N(t)$  follows integral of  $I(t)$

$$\text{final point } \Delta N \rightarrow N_t e^{-\frac{\Gamma_{in}}{\Gamma_s}} \quad \Gamma_{in} = \text{input fluence}$$

$$\text{check ansatz compare } \frac{1}{\tau_p} \frac{\Gamma_{in}}{\Gamma_s} \cdot N_t e^{-\frac{\Gamma_{in}}{\Gamma_0}} \text{ to } \frac{N_t}{\tau}$$

now it depends on what  $\Delta N_{final}$  is