Physics 311: Homework 3, due Tuesday 9/19/06 end of day.

1 Problems from Boas

Problems 5.9, 5.22, 10.5, 10.30

2 Free vibrations of the damped oscillator

In this problem we consider the damped harmonic oscillator that carries out a free motion, i.e., there is no external force acting. In this case Newton's law is given by

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0 \ . \tag{1}$$

In this problem we seek solutions of the form

$$x(t) = C\exp(-i\omega t) \ , \tag{2}$$

where C is a real constant.

Problem a: Insert this solution in the equation of motion and show that the angular frequency satisfies

$$\omega^2 + i\omega\gamma - \omega_0^2 = 0 \ . \tag{3}$$

Problem b: Show that the angular frequency of the vibration satisfies

$$\omega = -\frac{i\gamma}{2} \pm \sqrt{\omega_0^2 - \gamma^2/4} \tag{4}$$

Problem c: Consider the case $\gamma = \sqrt{3}\ \omega_0$. Plot the allowed values of ω in the complex plane, and sketch $x(t)$. Remember that C is real, and don't forget to take the real part of expression (2).

Problem d: Do the same proble for $\gamma = \sqrt{5}\,\omega_0$.

Problem e: The solution of problem c is oscillatory, this corresponds to he weakly damped case. The solution of problem d corresponds to the oscillator that is damped so strongly that it does not oscillate. In both cases the angular frequency has two possible vales. There is an intermediate value of γ for which only one value of ω is allowed. Find the value of γ for this case (called the *critically* damped oscillator).

Problem e: Do problem c for the critically damped oscillator.

3 Forced vibrations of the damped oscillator

Problem a: Derive the amplitude and phase spectrum for a damped harmonic oscillator (equation 1) driven by a force: $F(t) = F_0 e^{i\omega t}$.

Problem b: Obtain an analytic expression for the frequency at which the amplitude is a maximum and for the value of the amplitude at this frequency.

Problem c: How is the damping constant γ related to the full-width at half maximum (FWHM) of the amplitude spectrum? (We want a formula, here.)