Phys 361 Homework 2

1) (based on Problem 3.24 in Pollack & Stump)

Take an electric field of the form

$$\vec{E}(\vec{x}) = (2x^2 - 2xy - 2y^2)\hat{\imath} + (-x^2 - 4xy + y^2)\hat{\jmath}$$

a) Does this field have curl? If it doesn't, find a potential function that corresponds to it.

b) In either case, find the divergence of the field. In plain English, what's the difference between an electric field that has divergence and one that doesn't? There's definitely more than one way to answer that.

2a) There's a surprising theorem in electrostatics called Earnshaw's theorem. It basically states that you can't take a charge and hold it in a *stable* equilibrium using only a collection of static charges. Everyone's first response when they hear this theorem is "No way, all I gotta do is… wait, no, that doesn't work… maybe if I… etc." So get it out of your system. Take a positive point charge and do your best to construct a static system of charges that holds said positive point charge in a stable equilibrium without touching it. You will not succeed, but that's okay. Trying to break stuff is a good way to learn about stuff. Write up a couple of your more convincing attempts (that is, arrangements where it was pretty hard to see why they failed).

b) Proving Earnshaw's theorem formally is non-trivial, but you can argue for its correctness informally. Do so. First tell me what the divergence of *E* would have to look like in a location that is a stable equilibrium for a positive point charge. Then look at Gauss's law and tell me the issue we seem to have.

c) Does Earnshaw's theorem hold if we restrict ourselves to a 2-D plane in 3D space? How about if we're in a legitimately 2-D universe?

3) (based on 3.37 in Pollack & Stump)

Starting from the electric potential for a pointlike dipole with dipole moment $\vec{p}$:

$$V(\vec{x}) = \frac{\vec{p} \cdot \hat{r}}{4\pi\varepsilon_0 r^2}$$

derive the expression for the electric field of said dipole.

4) In class, we worked out the multipole expansion for the potential of a system of two charges q_1 and q_2, distances r_1 and r_2 from the origin. For $r \gg r_1, r_2$, the first three terms of the expansion looked like:

$$V(r,\theta) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2}{r} + \frac{(q_1 d_1 + q_2 d_2)\cos\theta}{r^2} + \frac{(q_1 d_1^2 + q_2 d_2^2)}{2r^3}(3\cos^2\theta - 1)\right) \qquad (1)$$

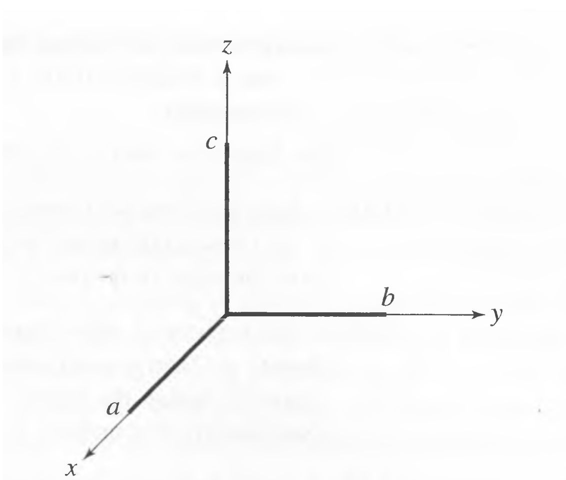
I also claimed that if we generalized things to systems with lots of charge, the multipole potential out to the quadrupole term would look like:

$$V(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{\hat{r}\cdot\vec{p}}{r^2} + \frac{\hat{r}\cdot\overleftrightarrow{Q}\cdot\hat{r}}{r^3}\right) \qquad (2)$$

where q is the monopole moment, $\vec{p}$ is the dipole moment vector, and $\overleftrightarrow{Q}$ is the quadrupole moment tensor. (Note to Griffiths readers: He uses the same form, but reserves it for one of the end-of-chapter problems instead of the main text)

I don't consider it entirely obvious that equation (1) follows from the (2), so let's grind it out and get a little practice with the machinery. Start with (2) and the same system of two point charges q_1 and q_2 from above. Use the definitions of q, $\vec{p}$, and $\overleftrightarrow{Q}$ to show that you can get (1) directly from (2), though we originally derived (1) via other means.

5) (based on problem 3.35 in Pollack & Stump)



Here's a system made of three uniformly charged rods of lengths a, b, and c, each with charge density λ. Each rod starts at the origin and extends along one of the Cartesian axes. At first glance this looks like a pretty gnarly system, but describing it in terms of multipole moments is actually not too bad.

a) First, find the dipole moment of this charge distribution.

b) Then, find the first two terms of the multipole expansion of the potential on the axis, $V(0,0,z)$, for $z \gg a, b, c$.

c) Now find the monopole and dipole pieces of $\vec{E}$ on the z axis, for $z \gg a, b, c$. Be careful here! If you just start taking derivatives of the answer from (b), you might lose information on account of us having already gotten rid of the x and y variables in that expression.

6) (based on problem 3.43 in Pollack & Stump)

The potential function for an atom is a little bit complicated on account of an atom being a composite of negative electrons and a positive nucleus. One decent approximation involves the kind of $\frac{1}{r}$ dependence that you'd expect for a point charge, modified by an exponential that represents the manner in which the oppositely charged bits screen each other out a little:

$$V(r) = \frac{Ze}{4\pi\varepsilon_0 r} e^{-\frac{r}{a}}$$

where a represents an effective atomic radius, and Z is the atomic number.

a) Find the charge density $\rho(r)$ that would produce the above voltage function. Be careful – you should end up with a term that represents a nucleus and one that represents an electron cloud. If you only got one term, you may have zeroed out something that you shouldn't have.

b) Sketch a graph of ρ vs r.

c) Show explicitly that the total charge in the system is zero, as it should be.