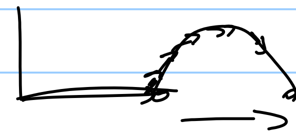
Lecture 39

Note Title 4/28/2006

$\int d\vec{l}$

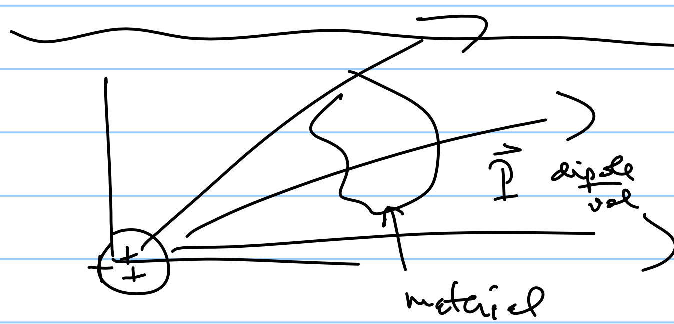
$\shortparallel$

$\int r\,d\theta\,\hat{\theta}$



$\vec{l}_f - \vec{l}_i$

$$\int |d\vec{l}| = \int |r\,d\theta\,\hat{\theta}| = \int r\,d\theta\,\underbrace{|\hat{\theta}|}_{1} = \int_0^{2\pi} r\,d\theta = 2\pi r$$



- permanent electric dipole (thermal distribution $\vec{P}$)
- induced dipole

Solved by find $\sigma_b = \vec{P}\cdot\hat{n}$ $\rho_b = -\vec{\nabla}\cdot\vec{P}$

$V = \int \frac{k\,dq}{r}$ $dq = \rho_b\,d\tau$ or $\sigma\,da$

magnetic materials $\vec{M}$ $\frac{\text{dipole moment}}{\text{vol}}$ mag dipole mom

$\Rightarrow$ $\vec{K}_b = \vec{M} \times \hat{n}$ $\quad \vec{J}_b = \vec{\nabla} \times \vec{M}$

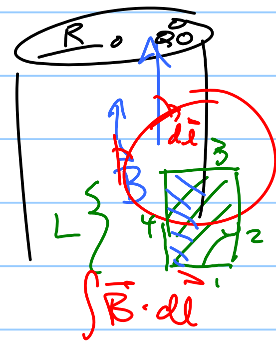
$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}_b}{r} d\tau + \frac{\mu_0}{4\pi} \int \frac{K_b \, d\vec{a}}{r}$$

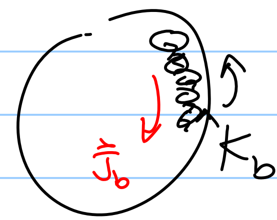
$= (M \times \hat{n}) |d\vec{a}|$

$\vec{M} = kr\,\hat{z}$

$\vec{J}_b = -k\,\hat{\varphi}$

$\vec{K}_b = kR\,\hat{\varphi}$





$$\oint \vec{B} \cdot d\vec{l} = \int_1 + \int_2 + \int_3 + \int_4 B\, dl \cos 0$$

$= BL = \mu_0 I_{enc}$

$$I_{enc} = \int_0^L K_b \, dl + \int J_b \, da$$

(current/length; const)

$KL + J(R-r)L$

1.) Hard to find $\vec{M}$

2.) In an expt. we measure free current

$$\vec{J} = \vec{J}_b + \vec{J}_f \qquad \frac{1}{\mu_0}\vec{\nabla}\times\vec{B} = \vec{J} = \vec{J}_b + \vec{J}_f$$

($\vec{J}_b$: $\vec{\nabla}\times\vec{M}$)

$$\vec{\nabla}\times\underbrace{\left(\frac{\vec{B}_0}{\mu_0} - \vec{M}\right)}_{\vec{H}} = \vec{J}_f$$

← what we measure

$\vec{\nabla}\times\vec{H} = \vec{J}_f$ but that doesn't uniquely specify $\vec{H}$

need $$\vec{\nabla}\cdot\vec{H} = \vec{\nabla}\cdot\left(\frac{\vec{B}_0}{\mu_0} - \vec{M}\right) = \frac{1}{\mu_0}\underbrace{\vec{\nabla}\cdot\vec{B}}_{0} - \vec{\nabla}\cdot\vec{M}$$

$$\int \vec{\nabla}\cdot\vec{M}\, d\tau = \oint \vec{M}\cdot d\vec{a}$$

$\vec{\nabla}\cdot\vec{M} \neq 0$ in general

$\vec{\nabla}\cdot\vec{H} \neq 0$ " "

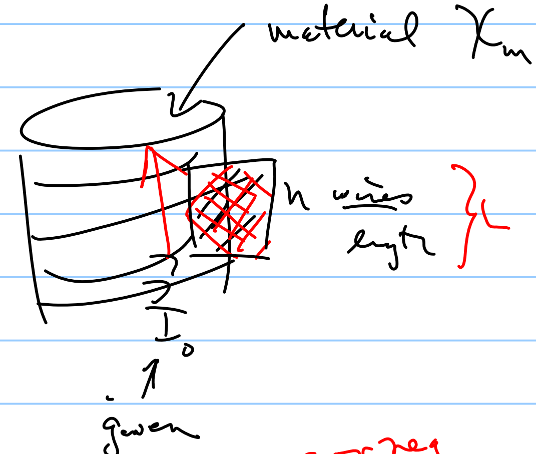
① Hard to find $\vec{M}$

assume linear approx $\vec{M} = \chi_m \vec{H}$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu_0} - \chi_m \vec{H}$$

$$\vec{B} = \underbrace{\mu_0(1+\chi_m)}_{\mu}\vec{H} = \mu \vec{H}$$

Ex: solenoid



$$\vec{\nabla} \times \vec{H} = \vec{J}_f$$

$$\int \vec{\nabla} \times \vec{H} \cdot d\vec{a} = \oint \vec{H} \cdot d\vec{l}$$

$$\underbrace{\int \vec{J}_f \cdot d\vec{a}}_{n I_0 \cancel{L}} = \oint \vec{H} \cdot d\vec{l}$$

$$n I_0 \cancel{L} = H \cancel{L}$$

$$\vec{B} = \mu_0(1+\chi_m) H = \underbrace{\mu_0(1+\chi_m)}_{\mu} n I_0$$