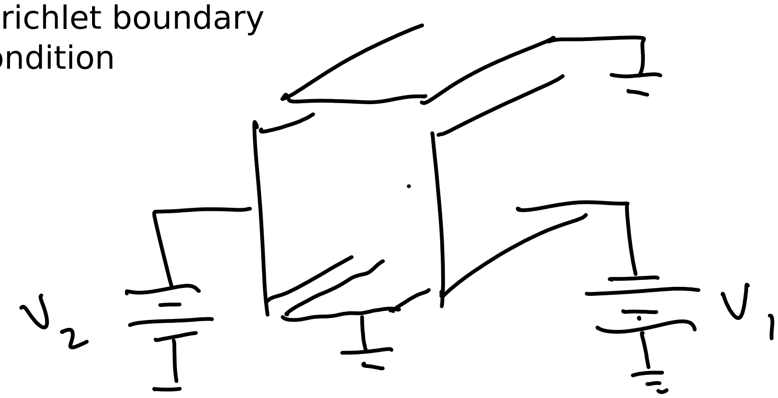
Lecture 40

Last week of classes: PDE boundary conditions applied to E and B, magnetic circuits, design problem, and possibly separation of variables. Homework due Wednesday April 30.

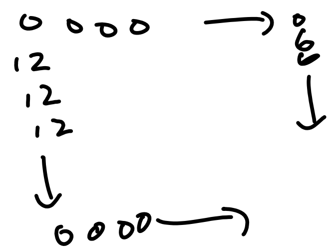
View text on PDE's to demonstrate Dirichlet and Neuman boundary conditions.

Examples:

Dirichlet boundary condition



The voltage is specified on the surfaces

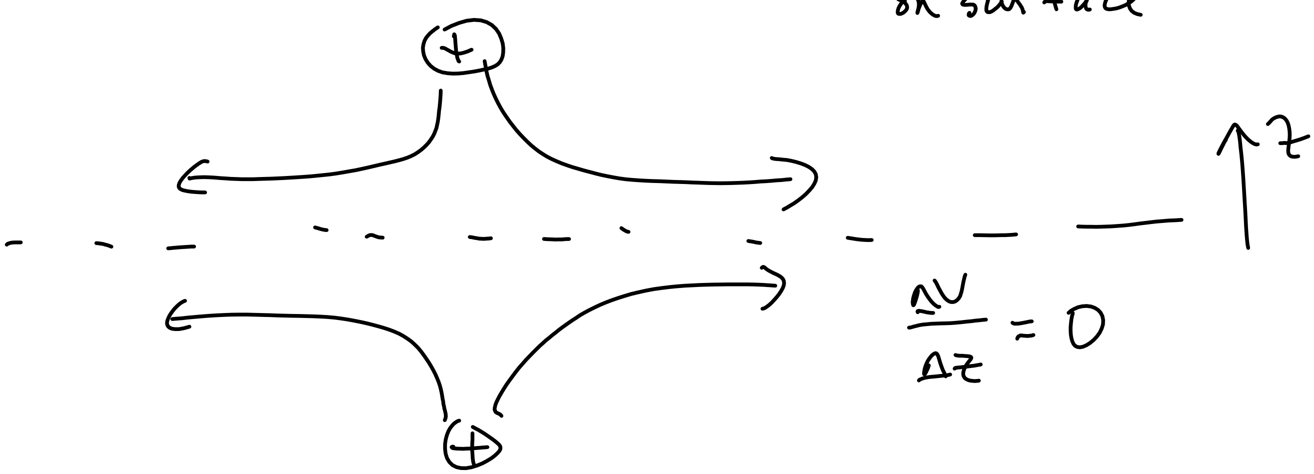


Relaxation method

Neuman boundary condition

$$\vec{E}_n = -\vec{\nabla} V_n = 0$$

on surface



$$\frac{\Delta V}{\Delta z} = 0$$

Mixed boundary conditions combine Neuman on part of the boundary and Dirichlet on another part (your hmwk problem is like this)

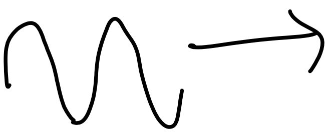
Final Homework problem: A spinless non-zero rest mass particle in an eigenstate of energy is incident on a massive mirror moving at constant speed. It reflectes elastically. Show that probability is conserved.

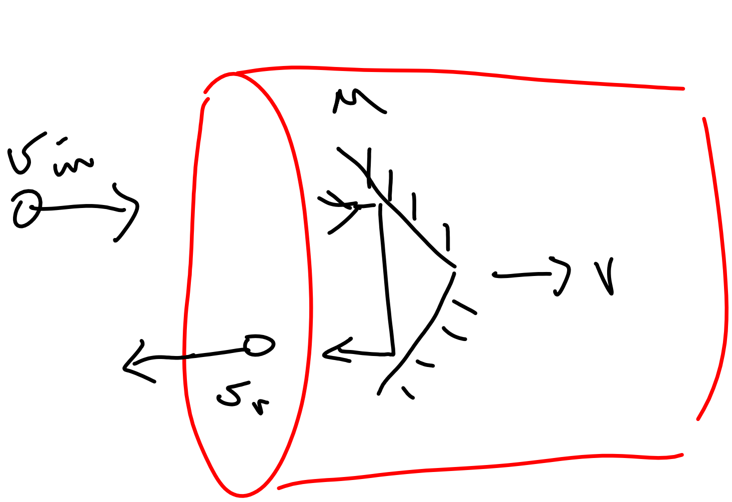
Look up probability current on wikipedia.

I will give you the solns to the Schrodinger equation for eigenstates of energy incident and reflecting from a mirror consisting of an infinite barrier. You need to use these in the integral form of the conservation law to show it is satisfied (LHS=RHS).

To eliminate issues of interference between the incident and reflected beams, let the mirror (corner reflector) displace the reflected beam.
Look up corner reflector on wikipedia.

$$\psi_{in} = e^{i(k_{in}x - \omega_{in}t)}$$





$$\psi_{ref} = e^{i(-k_r x - \omega_r t + \delta)}$$

δ is a phase shift on reflection

$$\hbar k_{in} = p_{in} = mv$$

$$\hbar k_r = p_r = mv_r$$

Conservation of momentum and energy give

$$mv + MV = mv_r + MV_r$$

$$\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \frac{1}{2}mv_r^2 + \frac{1}{2}MV_r^2$$

2 eqns in 2 unknowns (v_r, V_r)

Note that the amplitude is one for both incident and reflected wavefunctions.

How would you generate large currents and magnetic fields?

Why would you generate large currents and magnetic fields?

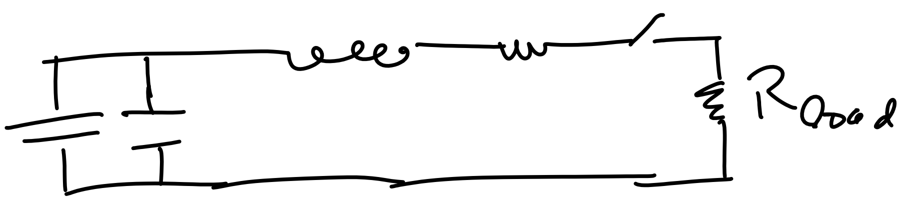
Verify models of how material behaves at large B. Generate EM radiation.

One way is to store energy in a capacitor bank and then discharge it.

Questions:

incongruous: How could you get all that energy out in a short time? Doesn't the capacitor have inductance which would limit the extraction time?

Answer:



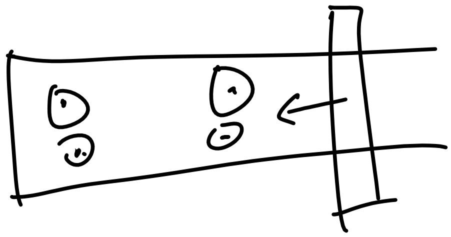
$$V_c - V_L - IR = 0 \qquad V_L = -L\frac{dI}{dt}$$

The inducatance comes from the winding of the capacitor plates.

Look up low inductance capacitors and resistors.

causal/creative: Could you use Faraday's law as in an electric motor to generate large currents?

causal/creative: What's the simplest example of such a motor?



Faraday's and Lenz's laws try to maintain the flux as the rod move in by generating an EMF which produces a current which produces a B to keep the flux constant.

Look up flux compression on wikipedia.

Questions:

-informational: Is this flux compressor what was used in "Back to the future?"
Ans: no that was a flux capacitor (there is of course no such thing).

-congruous: How do I calculate conservation of energy?

Comments on the metal detector design problem.

Look up the "engineering design process."

Define the problem: To build a EM sensor to detect small buried metal objects.

Background reserach:

Define requirements: Lightweight, able to sense a penny a foot deep, inexpensive . . .

Generate ideas:

-Sense the eddy currents that generate a magnetic field.

-Sense the change in indutance of a sensor which is an inductor as it is moved over the ground.

-Sense the time delay of a EM pulse as it reflects from metal.

-measure the resistivity of the soil with probes (like the Titanic exercise).

-measure the capacitance between a probe and the surface of the Earth.

-differential detection: one loop going cw with sample and other ccw without to cancel external fields.

-AC magnetic field to induce eddy currents even if the sensor is not moving.

-measure the force on a strong magnet swept over the surface.

Generate ideas (unrestrained) then do some positive and negative critical thinking afterwards.

Example: putting two wires in the Earth and measuring the current may seem impractical. However, ground penetrating radar needs to be touching the surface to mitagate surface reflections. Look at the image on wikipedia which shows a wheeled device moving along the surface.

You don't need to know everything about the subject before you can contribute to new knowledge. Being aware of this encourages you to think of novel ideas rather than waiting until you know it all (you'll never be there).

Example the axe which has been around for thousands of years.
Look up "Physics-exploiting axe splits wood in record time."

Develop a decision matrix or rubric.

Build a prototype.

Test and redesign.

Group project for participation credit: Design an EM device which shoots a projectile.

Motivation:
-collide two materials to generate nuclear fusion
-launch material into orbit
-new type of gun