Magnetostatics

Note Title 3/26/2007

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

$$\underbrace{\int \vec{\nabla}\times\vec{B}\cdot d\vec{a}} = \int \mu_0 \vec{J}\cdot d\vec{a}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$$ Amps Law

Electrostatic

$\vec{\nabla}\times\vec{E} = 0$

for B

$\vec{\nabla}\times\vec{B} \neq 0$

$\vec{\nabla}\times\vec{\nabla}V = 0$ ALWAYS

↑ scalar function

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} & \frac{\partial V}{\partial z} \end{vmatrix}$$

To define a vector function we need $\vec{\nabla}\times$ & $\vec{\nabla}\cdot$

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ $\qquad$ $\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\cdot(\underbrace{\vec{\nabla}\times\vec{A}}) = 0$ ALWAYS for any $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$

$$\boxed{\vec{B} = \vec{\nabla}\times\vec{A}}$$ defn of $\vec{A}$

Can get same $\vec{B}$ for diff $\vec{A}$'s

- adding a constant to A
- add $\vec{\nabla}\psi$ $\qquad$ $\vec{A} = \vec{A}' + \vec{\nabla}\psi$

↑ scalar function

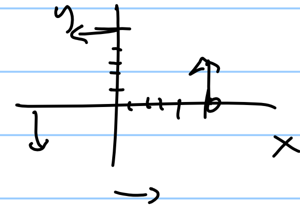
$$\vec{B} = \vec{\nabla}\times\vec{A} = \vec{\nabla}\times(\vec{A}' + \vec{\nabla}\psi) = \vec{\nabla}\times\vec{A}' + \underbrace{\vec{\nabla}\times\vec{\nabla}\psi}_{\text{ALWAYS } \emptyset}$$

We can choose ψ (choice of gauge)

$\vec{\nabla}\cdot\vec{A} = \vec{\nabla}\cdot\vec{A}' + \vec{\nabla}\cdot\vec{\nabla}\psi$ $\quad$ choose $\vec{\nabla}\cdot\vec{\nabla}\psi$ such that $\vec{\nabla}\cdot\vec{A} = 0$

$\boxed{\vec{\nabla}\cdot\vec{A} = 0}$ Coulomb guage + $\vec{B} = \vec{\nabla}\times\vec{A}$

Ex: $\vec{A} = -\frac{B_0 y}{2}\hat{x} + \frac{B_0 x}{2}\hat{y} + 0\hat{z}$



find $\vec{B} = \vec{\nabla}\times\vec{A}$

$$\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{B_0 y}{2} & \frac{B_0 x}{2} & 0 \end{vmatrix} = B_0\hat{z}$$

OR $\vec{A} = -B_0 y\hat{x}$

$$B = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -B_0 y & & \end{vmatrix} = B_0\hat{z}$$

What current density produces constant azimuthal potential?

cylindrical coords

$\vec{A} = k\hat{\varphi} + \emptyset\hat{s} + \emptyset\hat{z}$

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{1}{r}\frac{\partial(rk)}{\partial r}\hat{z} = \frac{k}{r}\hat{z} + \emptyset\hat{\varphi} + \emptyset\hat{s}$

$\vec{J} = \frac{1}{\mu_0}\vec{\nabla}\times\vec{B} = \frac{1}{\mu_0}\left(-\frac{\partial}{\partial r}\left(\frac{k}{r}\right)\right)\hat{\varphi} = \frac{k}{\mu_0 r^2}\hat{\varphi}$

Note $\vec{J}$ $\hat{\varphi}$ direction and $\vec{A}$ points in $\hat{\varphi}$

Explain in words how you would find the vector potential from an infinite wire carrying constant current.

get B from Amps law then use $\vec{B} = \vec{\nabla}\times\vec{A} \Rightarrow$ PDE

Given $\vec{J}$ find $\vec{A}$

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ OR $\underbrace{\vec{\nabla} \times \vec{\nabla} \times \vec{A}} = \mu_0 \vec{J}$

Vector Laplacian

$\vec{\nabla}(\underbrace{\vec{\nabla} \cdot \vec{A}}_{\emptyset}) - \nabla^2 \vec{A}$

Coulomb Gauge

Cartesian coords : 3 scalar laplacian

$\nabla^2 A_x = -\mu_0 J_x$ $\quad \nabla^2 A_y = -\mu_0 J_y$ $\quad \nabla^2 A_z = -\mu_0 J_z$

$\updownarrow$

$\nabla^2 V = -\rho/\epsilon_0$ $\qquad V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,')\,d\tau'}{|\vec{r} - \vec{r}\,'|}$

$A_x(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{J_x(\vec{r}\,')\,d\tau'}{|\vec{r} - \vec{r}\,'|}$ Given $\vec{J}$ find $\vec{A}$