Homework 2 due 7 Sept. 2007 in class

PH462 EM Waves and Optical Physics posted: 31 Aug 2007

1. The case of wave propagation in spherical symmetry is important to describe focusing and diverging beams.
   a. Start with the scalar wave equation $\nabla^2\psi = \frac{1}{\mathrm{v}^2}\frac{\partial^2\psi}{\partial t^2}$, and assume angular symmetry (not true in general) so that $\psi = \psi(r,t)$. Write the wave equation in spherical coordinates, and that $\psi(r,t) = \frac{1}{r}\exp[i(k_0 r \pm k_0 \mathrm{v}t)]$ satisfies this equation. Which sign corresponds to an outward propagating wave?
   b. A diverging beam that is propagating along the z-axis can be thought of as a section of a spherical wave. Using a Taylor expansion for $z \gg x$ and $z \gg y$, show that the spherical wave can be written in the following approximate form (the paraxial approximation): $\frac{1}{r}\exp[ik_0 r] \approx \frac{1}{z}\exp[ik_0 z]\exp\left[\frac{ik_0\left(x^2+y^2\right)}{2z}\right]$
   c. Make a sketch of the wavefronts (lines of constant phase) for this wave.

2. Consider an electromagnetic plane wave, with field amplitude E_0 traveling in vacuum with a wavevector magnitude k_0 at an angle of $+65^{\circ}$ to the *z*-axis. The **k**-vector is in the *y*-*z* plane and the *E*-field direction is along the *x*-axis.
   a. Write an expression in Cartesian coordinates for wave, making sure to define the polarization direction and the vector direction of **k**. Use complex exponential notation.
   b. Use one of the Maxwell equations to calculate the *B*-field for the wave in part a. Make a sketch of the directions of **E**, **B** and **k**.

3. Consider a wave with the Jones vector $\hat{\mathrm{E}}_1 = \begin{pmatrix} -i \\ 2+i \end{pmatrix}$. What type of polarization is this wave? Find the Jones vector $\hat{\mathrm{E}}_2$ that is orthogonal to this vector (so that $\hat{\mathbf{E}}_1^* \cdot \hat{\mathbf{E}}_2 = 0$).

4. Demonstrate using the Jones vector notation that right- and left-circularly polarized light waves are orthogonal.

5. A wave passes through a horizontal polarizer and exits in the horizontal polarized state. If we place a vertical polarizer next, so that these polarizers are "crossed", no light gets through. If we insert a polarizer oriented at 45° between the H- and V- polarizers, how much light gets through. Show how you calculate the result.