11-14-07 Review Part I

Note Title 11/14/2007

ON $[-\ell,\ell]$ $\cos(\pi x/\ell)$

$= \cos(2\pi x/2\ell)$

as x goes from $-\ell$ to ℓ
arg. of cos goes from

$-\pi$ to $+\pi$ } 2π period

ON $[0, L]$ $\cos(2\pi x/L)$

as x goes from 0 to L
arg of cos goes from

0 to 2π } 2π period

So on $[0, L]$

$$f(x) = \frac{a_0}{2} + \sum_{N=1}^{\infty} a_N \cos\left(\frac{2\pi N x}{L}\right)$$

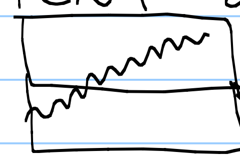
$$+ \sum_{N=1}^{\infty} b_N \sin\left(\frac{2\pi N x}{L}\right)$$

Review

Big ideas
- Fourier Transforms
- Fourier Series
- Discrete Fourier Trans.
  (FFT)

  when to apply each

Applications
- Frequency content of data

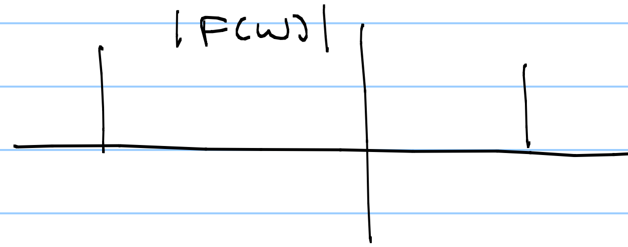


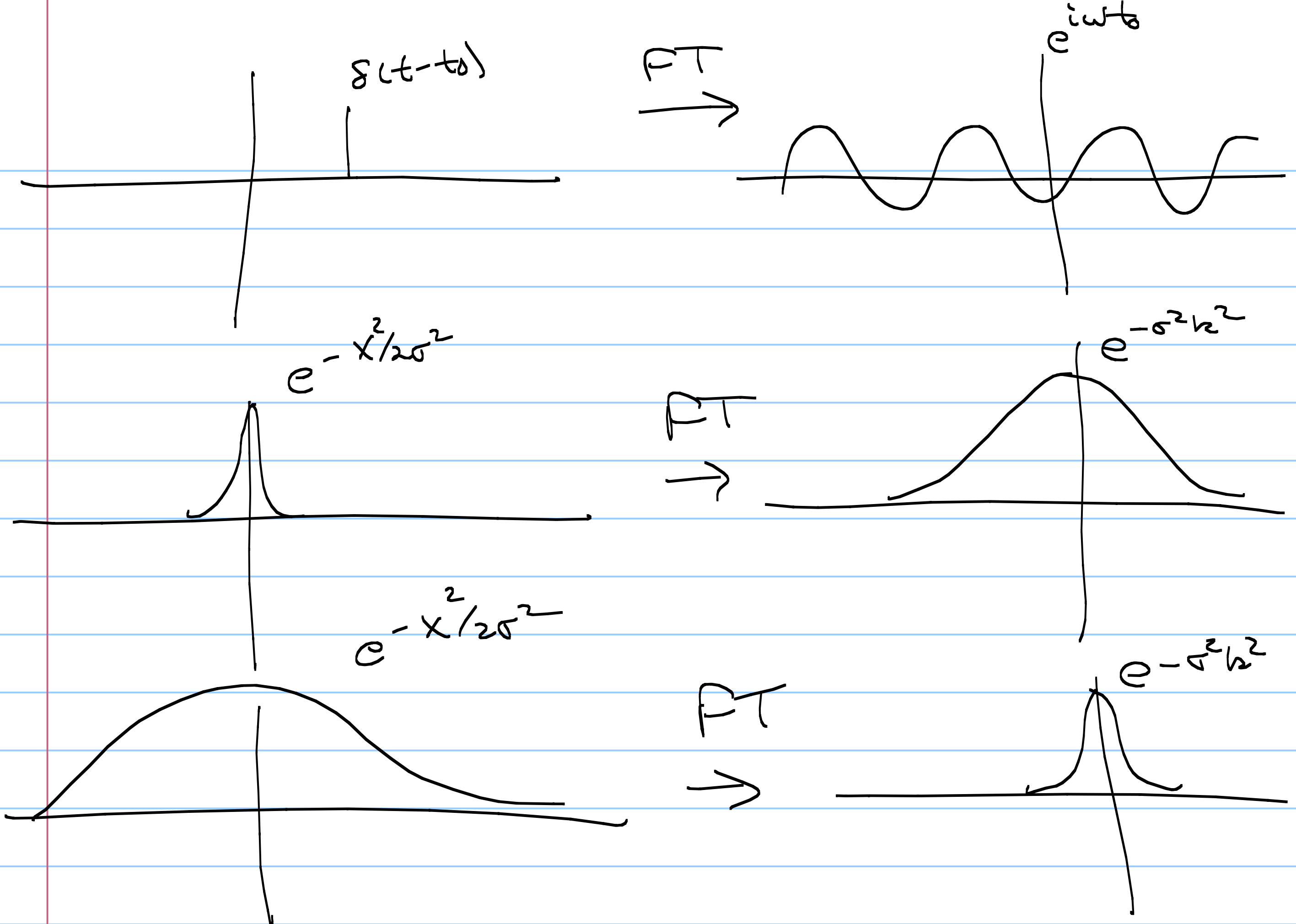
Be able to do this by inspection for simple data sets

- Solving differential equations

Time – Frequency trade off

$f(t)$ $|F(\omega)|$





I will give you a simple FT, DFT, and 1 or 2 terms of a FS to compute

Eg. $x = \{0, 0, 1\} \longrightarrow \text{DFT} = ?$

$$C_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} f_n \, e^{-2\pi i n k/N}$$

$$f(t) = e^{-|x|} \qquad \text{FT or FS}$$

more Big ideas

Sampling
Sampling rate
Nyquist frequency
Sampling theorem
Aliasing
Band limited functions

$$\delta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{it\omega}\, d\omega$$

$$\Rightarrow \sqrt{2\pi}\,\delta(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} 1\, e^{i\omega t}\, d\omega$$

$$w = -\omega$$
$$d\omega = -dw$$

$$\sqrt{2\pi}\,\delta(t) = \frac{1}{\sqrt{2\pi}}\int_{+\infty}^{\infty} 1\, e^{-iwt}\,(-1)\, dw$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} 1\, e^{-iwt}\, dw$$

$$\Rightarrow FT[1] = \sqrt{2\pi}\,\delta(t)$$