FOWLES "MODERN OPTICS"

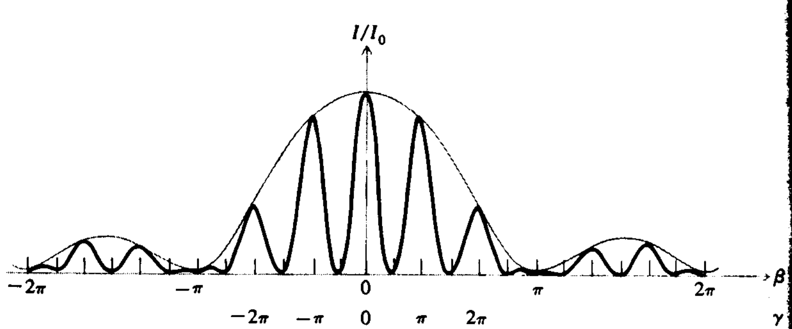

Figure 5.16. Fraunhofer diffraction pattern of a double-slit aperture.

Multiple Slits. Diffraction Gratings Let the aperture consist of a grating, that is, a large number N of identical parallel slits of width b and separation h (Figure 5.17). The evaluation of the diffractional integral is carried out in a manner similar to that of the double slit:

$$\begin{aligned}\int_{\mathscr{A}} e^{iky\sin\theta}\,dy &= \left[\int_0^b + \int_h^{h+b} + \int_{2h}^{2h+b} + \cdots + \int_{(N-1)h}^{(N-1)h+b}\right] e^{iky\sin\theta}\,dy \\ &= \frac{e^{ikb\sin\theta}-1}{ik\sin\theta}\left[1 + e^{ikh\sin\theta} + \cdots + e^{ik(N-1)h\sin\theta}\right] \\ &= \frac{e^{ikb\sin\theta}-1}{ik\sin\theta}\cdot\frac{1-e^{ikNh\sin\theta}}{1-e^{ikh\sin\theta}} \\ &= be^{i\beta}e^{i(N-1)\gamma}\left(\frac{\sin\beta}{\beta}\right)\left(\frac{\sin N\gamma}{\sin\gamma}\right)\end{aligned} \tag{5.29}$$

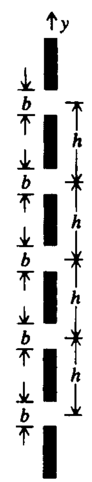

Figure 5.17. Multiple-slit aperture or diffraction grating.

where $\beta = \frac{1}{2}kb\sin\theta$ and $\gamma = \frac{1}{2}kh\sin\theta$. This yields the following intensity distribution function:

$$I = I_0\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\gamma}{N\sin\gamma}\right)^2 \tag{5.30}$$

The factor N has been inserted in order to normalize the expression. This makes $I = I_0$ when $\theta = 0$.

Again the single-slit factor $(\sin\beta/\beta)^2$ appears as the envelope of the diffraction pattern. Principal maxima occur within the envelope at $\gamma = n\pi$, $n = 0, 1, 2, \ldots$, that is,

$$n\lambda = h\sin\theta \tag{5.31}$$

which is the grating formula giving the relation between wavelength and angle of diffraction. The integer n is called the *order of diffraction.*

Secondary maxima occur near $\gamma = 3\pi/2N$, $5\pi/2N$, and so forth, and zeros occur at $\gamma = \pi/N$, $2\pi/N$, $3\pi/N$. . . . A graph is shown in Figure 5.18(a). If the slits are very narrow, then the factor $\sin\beta/\beta \approx 1$. The first few primary maxima, then, all have approximately the same value, namely, I_0.

Resolving Power of a Grating The angular width of a principal fringe, that is, the separation between the peak and the adjacent minimum, is found by setting the *change* of the quantity $N\gamma$ equal to π, that is, $\Delta\gamma = \pi/N = \frac{1}{2}kh\cos\theta\,\Delta\theta$, or

$$\Delta\theta = \frac{\gamma\lambda}{Nh\cos\theta} \tag{5.32}$$

Thus if N is made very large, then $\Delta\theta$ is very small, and the diffraction pattern consists of a series of sharp fringes corresponding to the different orders $n = 0, \pm1, \pm2$, and so forth [Figure 5.18(b), (c)]. On the other hand for *a given order* the dependence of θ on the wavelength [Equation (5.31)] gives by differentiation

$$\Delta\theta = \frac{n\,\Delta\lambda}{h\cos\theta} \tag{5.33}$$

This is the angular separation between two spectral lines differing in wavelength by $\Delta\lambda$. Combining Equation (5.32) and (5.33), we obtain the *resolving power* of a grating spectroscope according to the Rayleigh criterion, namely,

$$\mathrm{RP} = \frac{\lambda}{\Delta\lambda} = Nn \tag{5.34}$$

In words, the resolving power is equal to the number of grooves N multiplied by the order number n.