MATH348 - November 21, 2011 **NAME**:

Exam II - 50 Points

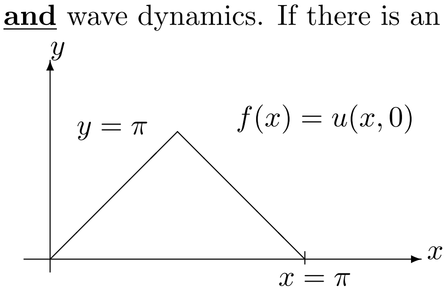
In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points) Conceptual Questions. For the following questions, assume that we are considering the physical problem on a bounded domain, $x \in (0, \pi)$.

   (a) Write down the heat **and** wave equations and any initial conditions needed for unique solutions. For each, what does the unknown function u measure?

   (b) Suppose we are given the boundary conditions $u_x(0, t) = 0$ and $u_x(\pi, t) = 0$ for each problem. Explain the physical meaning of **each** boundary condition for **both** the heat equation and wave equation.

   (c) The following graph gives the only nonzero initial configuration for each problem. Describe and/or draw the associated heat **and** wave dynamics. If there is an equilibrium state then be sure to state it.



2. (10 Points)

   (a) The following table contains different boundary conditions for the ODE, $F'' + \lambda F = 0$, $\lambda \in [0, \infty)$. Fill in each table element with either a yes or a no.

	Boundary value problem has a cosine solution	Boundary value problem has a sine solution	Boundary value problem has a nontrivial constant solution
$F'(0) = 0,\ F(L) = 0$			
$F(0) = 0,\ F(L) = 0$			
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   (b) Calculate the Fourier transform of $f(x) = \delta_{-2}(x) - \delta_2(x)$.

   (c) Noting that $\mathfrak{F}\{f'\} = -i\omega\mathfrak{F}\{f\}$, use Fourier transforms to find the general solution to $u_t = u_{xx}$ in the Fourier domain.

   (d) Find the value of λ such that $F'' + \lambda F = 0$ subject to $F(0) = 0$, $F'(\pi) = 0$ has a non-zero solution.

   (e) Assuming $u(x, t) = F(x, y)G(t)$, find the time ODE and space PDE consistent with $u_{tt} + u_t = u_{xx} + u_{yy}$.

3. (10 Points)

   (a) Show that $u(x,t) = \dfrac{1}{x-t}$ is a solution to $u_t + u_x = 0$.

   (b) Show that $u(x,t) = e^{it}\cos(x)$ is a solution to $iu_t = u_{xx}$.

4. (10 Points) Suppose that $u_t = u_{xx} + F(x,t)$ for $x \in (0,\pi)$ and such that $u_x(0,t) = u_x(\pi,t) = 0$. Find the ODEs associated with the time-dynamics of the previous PDE.

5. (10 Points) Find the unique solution to,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad \begin{matrix} x \in (0,\pi) \\ t \in (0,\infty) \end{matrix} \quad , \tag{1}$$

$$u(0,t) = 0,\ u(\pi,t) = 0, \tag{2}$$

$$u(x,0) = 0, \tag{3}$$

$$u_t(x,0) = g(x). \tag{4}$$

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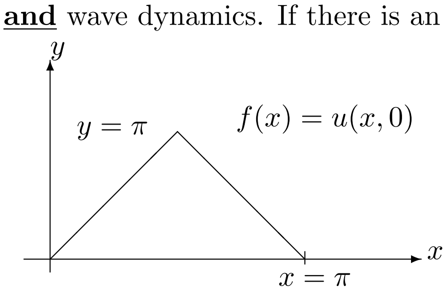

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(b) Calculate the Fourier transform of $f(x) = \delta_{-2}(x) + \delta_2(x)$.

(c) Noting that $\mathfrak{F}\{f'\} = -i\omega\mathfrak{F}\{f\}$, use Fourier transforms to find the general solution to $u_t = u_{xx}$ in the Fourier domain.

(d) Find the value of λ such that $F'' + \lambda F = 0$ subject to $F'(0) = 0$, $F(\pi) = 0$ has a non-zero solution.

(e) Given that $u_h(x,t) = \sum_{n=1}^{\infty} \sin(nx)e^{-n^2 t}$ is the homogeneous solution to $u_t = u_{xx} + F(x,t)$ where $F(x,t) = \sum_{n=1}^{\infty} f_n(t)\sin(nx)$, find the ODEs associated with the time dynamics of the general solution to the PDE.

3. (10 Points)

   (a) Show that $u(x,t) = \ln(x^2 + y^2)$ is a solution to $u_{xx} + u_{yy} = 0$.

   (b) Show that $u(r) = r^{-1}$ is a solution to $u_{rr} + 2r^{-1}u_r = 0$.

4. (10 Points) Using separation of variables, find the three ODEs consistent with the PDE $u_{tt} + u_t = u_{xx} + u_{yy}$.

5. (10 Points) Find the unique solution to,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad \begin{array}{c} x \in (0,\pi) \\ t \in (0,\infty) \end{array}, \tag{5}$$

$$u_x(0,t) = 0,\ u_x(\pi,t) = 0, \tag{6}$$

$$u(x,0) = f(x). \tag{7}$$

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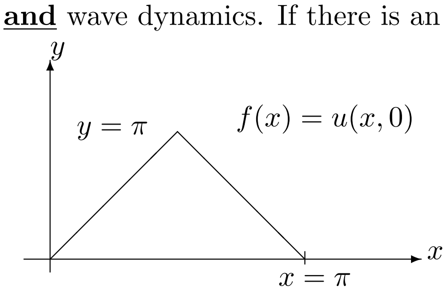

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(d) Assuming $u(x,t) = F(x,y)G(t)$, find the time ODE and space PDE consistent with $u_{tt} + u_t = u_{xx} + u_{yy}$.

(e) Given that $u_h(x,t) = \sum_{n=1}^{\infty} \sin(nx)e^{-n^2 t}$ is the homogeneous solution to $u_t = u_{xx} + F(x,t)$ where $F(x,t) = \sum_{n=1}^{\infty} f_n(t)\sin(nx)$, find the ODEs associated with the time dynamics of the general solution to the PDE.

3. (10 Points) Show that $u(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$ is a solution to $u_{xx} + u_{yy} + u_{zz} = 0$.

4. (10 Points) Given,

$$F'' + \lambda F = 0, \ \ \lambda \in [0, \infty)$$

Find the non-zero functions, F, and values for λ that satisfy the following boundary conditions.

(a) $F'(0) = 0$ and $F'(\pi) = 0$

(b) $F'(0) = 0$ and $F(\pi) = 0$

(c) $F(0) = 0$ and $F'(\pi) = 0$

5. (10 Points) Find the unique solution to,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad \begin{array}{c} x \in (0,\pi) \\ t \in (0,\infty) \end{array}, \tag{8}$$

$$u_x(0,t) = 0,\ u_x(\pi,t) = 0, \tag{9}$$

$$u(x,0) = 0, \tag{10}$$

$$u_t(x,0) = g(x). \tag{11}$$