E. Kreyszig, Advanced Engineering Mathematics, 9^{th} ed.	Section 8.1, pgs. 334-339
Lecture: Eigenvalues and Eigenvectors	Module: 07
Suggested Problem Set: {3, 5, 13, 14,16, 19, 21}	September 20, 2009

Quote of Lecture 7	
George Carlin: By and large, language is a tool for concealing the truth.	
	May 12, 1937 June 22, 2008

Okay, we know about $\mathbf{Ax} = \mathbf{b}$, or if we don't then we have some places to look. Now we concentrate on a special version of this equation where $\mathbf{b} = \lambda\mathbf{x}$, $\lambda \in \mathbb{C}$ and we say that,

$$\mathbf{Ax} = \lambda\mathbf{x}, \tag{1}$$

is an eigenvalue-eigenvector problem for the square matrix $\mathbf{A}_{n\times n}$. Specifically, $\mathbf{x}$ is called the eigenvector corresponding to the eigenvalue λ. If we think of $\mathbf{A}$ as a linear transformation then λ is a measure of the transformation in the $\mathbf{x}$-direction. The set of all eigenvectors and their corresponding eigenvalues then provides yet another characterization of the transformation defined by $\mathbf{A}$.

Solving (1) is a two part process:

- Calculate the characteristic equation from,

  $$\det(\mathbf{A} - \lambda\mathbf{I}) = 0,$$

  and find λ by solving for the roots of the polynomial. These roots are often called the spectrum of $\mathbf{A}$ and can be denoted at $\sigma(\mathbf{A})$.
- Determine a basis for the null space of,

  $$(\mathbf{A} - \lambda\mathbf{I}),$$

  by solving $(\mathbf{A} - \lambda\mathbf{I})\mathbf{x} = \mathbf{0}$. The basis vectors are eigenvectors associated with the particular λ used to calculate them. Sometimes, the collection of all eigenvectors is called an eigenbasis for $\mathbf{A}$. I

If the eigenbasis of a matrix forms a basis for $\mathbb{R}^n$ then many interesting properties can be deduced. If this occurs AND the matrix is self-adjoint, $\mathbf{A}^H = \mathbf{A}$, then one can show that the spectrum is purely real and that the eigenbasis forms an orthonormal basis for $\mathbb{R}^n$. [1]

Lecture Goals

- Understand how the concept of linear rescaling is related to eigenvalue-eigenvector problems.
- Use previous concepts of linear algebra to deduce a method for calculating eigenvalues and eigenvectors.

Lecture Objectives

- Derive auxiliary equations needed to calculate eigenvalues and eigenvectors.
- Summarize 2×2 theory.
- Calculate the eigenbasis of various 'instructive' matrices.

[1]This concept underpins the theoretical measurements of quantum particles and will be important in the study of physical PDE.