

**3.3.13** Prove that there is no integer  $x$  such that  $x^3 - 4x = 7$ .

**3.3.15** Prove that there do not exist three consecutive natural numbers such that the cube of the largest is equal to the sum of the cubes of the other two.

**4.1.9**

1. Determine the intersection and union of  $[2, 5]$  and  $[-1, \infty)$ .
2. Determine the intersection and union of  $[2, 5]$  and  $[3.4, \infty)$ .
3. Determine the intersection and union of  $[2, 5]$  and  $[7, \infty)$ .

Now let  $a$ ,  $b$  and  $c$  be real numbers with  $a < b$ .

4. Explain why the intersection of  $[a, b]$  and  $[c, +\infty)$  is either a closed interval, a set with one element, or the empty set.
5. Explain why the union of  $[a, b]$  and  $[c, +\infty)$  is either a closed ray or the union of a closed interval and a closed ray.

**4.2.12** Prove the following proposition:

For all sets  $A$ ,  $B$  and  $C$  that are subsets of some universal set, if  $A \cap B = A \cap C$  and  $A^c \cap B = A^c \cap C$ , then  $B = C$

**4.2.8** Let  $A$  and  $B$  be subsets of some universal set  $U$

1. Prove that  $A$  and  $B - A$  are disjoint sets
2. Prove that  $A \cup B = A \cup (B - A)$ .