

## Lienard-Wiechert Potentials

pot'l's from a pt. charge instead of p, J

$$\Phi(\vec{r}, t) = \int \frac{\rho(\vec{r}', t')}{|\vec{r} - \vec{r}'|} d^3 r'$$

$t'$  = local time

charge moves along path  $\vec{R}(t')$   
observation  $\vec{r}$

For a point charge, we want a  $\delta$  function to represent it

$$q \delta(\vec{r}' - \vec{r}_e(t_R)) d^3 x' \text{ in terms of spatial localization}$$

or

$$q \delta(t' - t_R) dt' \text{ for temporal localization.}$$

$$t_R = t - \frac{|\vec{r} - \vec{r}_e(t')|}{c} \text{ retarded time - depends on } t'$$

We need to change to a variable of integration to complete

$$t'' = t' - t_R(t') \text{ integral along path.}$$

$$= t' - t + \frac{|\vec{r} - \vec{r}_e(t')|}{c} \quad \vec{r} \text{ is fixed at P}$$

$t$  is fixed at observation time.

$$\frac{dt''}{dt'} = 1 + \frac{1}{c} \frac{d}{dt'} \left( \sum_i (x - x_{ei})^2 \right)^{1/2}$$

$$= 1 + \frac{1}{c} \sum_i \left( \frac{d}{dx_{ei}} |\vec{r} - \vec{r}_{ei}| \right) \frac{dx_{ei}}{dt'} \quad \text{chain rule}$$

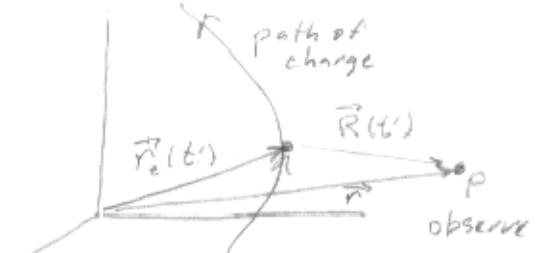
$$= 1 + \underbrace{\frac{1}{c} \nabla_{r_e} |\vec{r} - \vec{r}_e| \cdot \frac{d\vec{r}_e}{dt'}}_{-\frac{\vec{r} - \vec{r}_e}{|\vec{r} - \vec{r}_e|} = -\frac{\vec{R}}{R}} \rightarrow \vec{u} \text{ velocity of charge.}$$

$$\text{let } \vec{\beta} = \vec{u}/c$$

$$dt'' = dt' \left( 1 - \frac{\vec{\beta} \cdot \vec{R}}{R} \right)$$

or

$$dt' = \frac{R}{1 - \vec{\beta} \cdot \vec{R}/R} dt'' \quad \text{remember } R = R(t')$$



Now get potential

$$\Phi(\vec{r}, t) = q \int_{-\infty}^{\infty} \frac{\delta(t'')}{R(t'')} \frac{R(t')}{R(t') - \vec{B}(t') \cdot \vec{R}(t')} dt''$$

evaluate integrand at  $t''=0$

or  $t' = t - R(t')/c = \text{retarded time}$ ,

$$\rightarrow \Phi(\vec{r}, t) = \left. \frac{q}{R - \vec{B} \cdot \vec{R}} \right\}$$

similarly

$$\vec{A}(\vec{r}, t) = \left. q \left[ \frac{\vec{B}}{R - \vec{B} \cdot \vec{R}} \right] \right\}$$

L-W potentials

no longer integrals

$$\text{notation: } K = 1 - \vec{B} \cdot \vec{R}/R$$

$$\Phi = q \frac{1}{KR} \quad \vec{A} = q \left[ \frac{\vec{B}}{KR} \right]$$

this is still generally hard to evaluate:

$\Phi(t)$  depends on where the charge was  
time delay depends on distance.

picture reminding a movie:



$\Phi$  is increased if  $\vec{B}$  is toward P

$$K = 1 - \vec{B} \cdot \vec{R} > 0$$

## Lienard-Wiechert Fields

two methods:

- differentiate L-W potentials - Griffiths
- take limit of field equations for  $\rho, J$  - Hm

$$\Rightarrow \vec{E} = e \left[ \frac{(\vec{R} - \vec{\beta})(1 - \beta^2)}{K^3 R^2} + \frac{\vec{R} \times ((\vec{R} - \vec{\beta}) \times \vec{\alpha})}{c^2 K^3 R} \right] \quad \text{dts}$$

$$\vec{B} = [\vec{R}] \times \vec{E}$$

$$\vec{E} = \text{veloc term} + \text{accel term.}$$

veloc term: as  $\vec{\beta} = \vec{v}/c \rightarrow 0$      $\vec{E} = \frac{e \vec{R}}{R^2}$

$\vec{E}$  components  $\parallel$  to both  $\vec{R}$  and  $\vec{\beta}$

$\vec{B} \perp$  to  $\vec{E}$  and  $\vec{R}$

both  $\propto 1/R^2$

accel term:  $\propto \vec{\alpha}$  but  $\perp$  to  $\vec{A}$

$$E, B \propto 1/R$$

Calculation of  $\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B}$  will have 3 types of terms  $a, v$

$$S_{v,v} \propto E_v, B_v \propto 1/[R]^4$$

$$S_{av} \text{ and } S_{va} \propto 1/[R]^3$$

$$S_{aa} \propto 1/[R]^2$$

From enough above: only  $S_{aa}$  = radiation

Fields from a charge in uniform motion

- no acceleration

$$\vec{E} = \left[ \frac{e}{(KR)^3} (\vec{R} - \vec{R}\vec{\beta})(1 - \beta^2) \right]_{tr} \quad \text{with } K = 1 - \vec{\beta} \cdot \vec{R}$$

Can write  $\vec{E}$  in terms of present position, even though fields are retarded.

$$R_r = \bar{A}\bar{P} \quad R_p = \bar{C}\bar{P}$$

$$\bar{u}(t - t_r) = \bar{A}\bar{C} \rightarrow \bar{u}(t - t_r) + \vec{R}_p = \vec{R}_r$$

substitute for  $t_r$ :

$$t_r = t - R_r/c$$

$$\rightarrow \vec{R}_p = \vec{R}_r - R_r \vec{\beta} \quad \text{and} \quad \bar{A}\bar{C} = \beta R_r$$

Now find denominator

$$KR_r = R_r - \vec{\beta} \cdot \vec{R}_r = R_r - \beta R_r \cos \theta_r = \bar{B}\bar{P}$$

$$\bar{B}\bar{P}^2 = \bar{C}\bar{P}^2 - \bar{B}\bar{C}^2 = \bar{C}\bar{P}^2 - \bar{A}\bar{C}^2 \sin^2 \theta_r$$

$$(KR_r)^2 = R_p^2 - \beta^2 R_r^2 \sin^2 \theta_r$$

$$\text{common side } \bar{D}\bar{P} = R_r \sin \theta_r = R_p \sin \theta_p$$

$$\therefore (KR_r)^2 = R_p^2 - \beta^2 R_p^2 \sin^2 \theta_p$$

Finally,  $\vec{E} = \frac{e \vec{R}_p (1 - \beta^2)}{R_p^3 (1 - \beta^2 \sin^2 \theta_p)^{3/2}}$

since  $\vec{B} = \hat{R}_r \times \vec{E} = \frac{\vec{R}_r \times \vec{E}}{R_r} = \left( \frac{\vec{R}_p + \vec{\beta}}{R_p} \right) \times \vec{E} \quad \vec{E} \parallel \vec{R}_p$

$$\therefore \vec{B} = \vec{B} \times \vec{E}$$

observations:

- even though charge may be moving fast ( $\beta \rightarrow 1$ )  
field lines are along  $\vec{R}_p$  which point from present position
- angular dependence:  $\theta_{\text{p}} = \pi/2$   
 $\Rightarrow$  smallest near  $\pi/2$   
 $\Rightarrow$  bunching of field lines  $\perp \vec{B}$
- of course, no  $\vec{B}$  w/o velocity